International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 1 Issue 1 ‖ Aug. 2013‖ PP-19-21

On The Oscillatory Behavior of The Solutions to Second Order Nonlinear Difference Equations

Dr. B.Selvaraj[1], Dr. P. Mohankumar[2], A.Ramesh[3]
[1]*Dean of Science and Humanities, Nehru Institute of Engineering and Technology, Coimbatore, Tamilnadu, India.*
[2]*Professor of Mathematics, Aaarupadaiveedu Institute of Techonology, Vinayaka Mission University, Kanchipuram, Tamilnadu, India*
[3]*Senior Lecturer in Mathematics, District Institute of Education and Training, Uthamacholapuram, Salem-636 010, Tamilnadu India*

ABSTRACT: *In this paper we study ON THE OSCILLATORY BEHAVIOR OF THE SOLUTIONS TO SECOND ORDER NONLINEAR DIFFERENCE EQUATIONS of form*

$$\Delta\left(a_n\Delta y_n\right)+p_n f\left(y_{\sigma(n)}\right)=0, n\in N=\{0,1,2,...\}$$

KEYWORDS: *Oscillatory Behavior, second order, nonlinear difference equation*

I. INTRODUCTION

We are concerned with the Oscillatory behavior of the solutions of the Second order nonlinear difference equation of the form

$$\Delta\left(a_n\Delta y_n\right)+p_n f\left(y_{\sigma(n)}\right)=0, n\in N=\{0,1,2,...\} \qquad (1.1)$$

Where the following conditions are assumed to hold.

(C1): $\{a_n\},\{b_n\}$ and $\{\sigma(n)\}$ are positive sequence and $p_n \neq 0$ for infinitely many values of n

(C2): $\sigma(n)\leq n$ and $\lim_{x\to\infty}\sigma(n)=\infty$

(C3): $R_n=\sum_{s=n_1}^{n-1}\frac{1}{a_s}\to\infty$ as $n\to\infty$

(C4): $f:R\to R$ is continuous and $xf(x)>0$ for all $x\neq 0$ and $\frac{f(x)}{x}\geq L>0$

By Solution of equation (1.1) we mean a real sequence $\{y_n\}$ satisfying (1.1) for $n=0,1,2,3,...$ a solution $\{y_n\}$ is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise it is called Non-Oscillatory. The forward difference operator Δ is defined by $\Delta y_n = y_{n+1}-y_n$
Nowadays, much research is going in the study of Oscillatory behavior of the solutions of the Second order nonlinear difference equations (refer[1-9])

II. MAIN RESULTS

In this section we present some sufficient condition of the oscillation of all the solutions of equation (1.1)

Theorem 1

Assume the (H3) holds, $\Delta\sigma(n)\geq 0$ and

$$\sum_{s=n_2}^{\infty}\left(-LR_{\sigma(s)}p_s+\frac{a_{s+1}\left(\Delta R_{\sigma(s)}\right)^2}{4\left(s-n_1\right)R_{\sigma(s)}}\right)=\infty \quad \text{for } n\geq n_2 \qquad (1.2)$$

Then equation (1.1) is Oscillatory

Proof:

Let $\{y_n\}$ be non-oscillatory solution of equation (1.1) without loss of generality we may assume that $y_n > 0, y_{\sigma(n)} > 0$ and for $n \geq n_{1.}$ from equation (1.1) we have

$\Delta(a_n \Delta y_n) < 0$ for $n \geq n_{1.}$

Since $\Delta(a_n \Delta y_n)$ is non-increasing there exists a non negative constant k and $n_2 \geq n_{1.}$ such that $\Delta(a_n \Delta y_n) < -k$ for $n \geq n_2, k > 0$

Summing the last inequality from $n_1 to(n-1),$ we obtain

$a_n \Delta y_n \leq a_{n_1} \Delta y_{n_1} - k(n - n_1)$

Letting $n \to \infty$, we have

$a_n \Delta y_n \to -\infty$, thus, there is an integer $n \geq n_2, k > 0$

$a_n \Delta y_n \leq a_{n_1} \Delta y_{n_1} < 0$ that is $\Delta y_n \leq -l \dfrac{1}{a_n}$ summing the last inequality from $n_2 to(n-1),$

We have $y_n \leq y_{n_1} - l \sum_{s=n_1}^{n-1} \dfrac{1}{a_s}$

This implies that $y_n \to -\infty$ as $n \to \infty$, which contradiction to the fact that y_n is positive, Then $\Delta(a_n \Delta y_n) > 0$ and $a_n \Delta y_n > 0$

Define

$$\omega_n = \frac{R_{\sigma(n)} a_n \Delta y_n}{y_{(\sigma(n))}} > 0, \text{ then}$$

$$\Delta \omega_n = \frac{R_{\sigma(n)}}{y_{\sigma(n)}} \Delta(a_n \Delta y_n) + a_{n+1} \Delta y_{n+1} \Delta\left(\frac{R_{\sigma(n)}}{y_{\sigma(n)}}\right)$$

$$\Delta \omega_n = \frac{R_{\sigma(n)}}{y_{\sigma(n)}} \Delta(a_n \Delta y_n) + a_{n+1} \Delta y_{n+1} \left(\frac{\Delta R_{\sigma(n)}}{y_{\sigma(n+1)}}\right) + \frac{a_{n+1} R_{\sigma(n)} \Delta y_{n+1} \Delta y_{\sigma(n)}}{y_{\sigma(n)} y_{\sigma(n+1)}}$$

$$\Delta \omega_n = \frac{R_{\sigma(n)}}{y_{\sigma(n)}} (-p_n f(\sigma(n)) + \left(\frac{\Delta R_{\sigma(n)}}{R_{\sigma(n+1)}}\right) \omega_{n+1} + \frac{a_{n+1} R_{\sigma(n)} \Delta y_{n+1} \Delta y_{\sigma(n)}}{y_{\sigma(n)} y_{\sigma(n+1)}} \quad (1.3)$$

Consider

$$\Delta y_{\sigma(n)} = \Delta y_n + \sum_{n=1}^{n-1} \Delta y_n \geq (n - 1 - n_2) \Delta y_n; n \geq n_2$$

this implies that

$$\Delta y_{\sigma(n+1)} \geq (n - n_1) \Delta y_{n+1} \quad (1.4)$$

In view of (C2),(C4) equation (1.1) and (1.4) we get from equation (1.3) that

$$\Delta \omega_n \leq -L R_{\sigma(n)} p_n + \frac{\Delta R_{\sigma(n)}}{R_{\sigma(n+1)}} \omega_{n+1} - (n - n_1) a_{n+1} R_{\sigma(n)} \frac{(\Delta y_{n+1})^2}{(y_{\sigma(n+1)})^2}$$

That is

$$\Delta\omega_n \le -LR_{\sigma(n)} p_n + \frac{a_{n+1}\left(\Delta R_{\sigma(n)}\right)^2}{4\left(n-n_1\right)R_{\sigma(n)}} + \left[\sqrt{\frac{(n-n_1)R_{\sigma(n)}(w_{n+1})^2}{a_{n+1}(R_{\sigma(n+1)})^2}} - \sqrt{\frac{a_{n+1}(\Delta R_{\sigma(n)})^2}{4(n-n_1)\mathrm{R}_{\sigma(n)}}}\right]^2$$

This implies that

$$\Delta\omega_n \le \left(-LR_{\sigma(n)} p_n + \frac{a_{n+1}\left(\Delta R_{\sigma(n)}\right)^2}{4\left(n-n_1\right)R_{\sigma(n)}}\right)$$

Summing the Last inequality from $n_2 to (n-1),$ we have

$$\Delta\omega_n \le \omega_{n_1} - \sum_{s=n_2}^{\infty}\left(-LR_{\sigma(s)} p_s + \frac{a_{s+1}\left(\Delta R_{\sigma(s)}\right)^2}{4\left(s-n_1\right)R_{\sigma(s)}}\right)$$

Letting $n \to \infty$, we have, in view of (1.2) that $\omega_n \to -\infty$ as $n \to \infty$

Which contradicts $\omega_n > 0$ and the proof is complete

Example

The difference equations $\Delta\left(n\Delta y_n\right) + 4\left(2n+1\right)\left(y_{\sigma(n)}\right)^2 = 0; n > 2$ (1.5)

Satisfies all condition of theorem 1. Here $\sigma(n) = n-2$ and $f(x) = x^2$. Hence all solutions of the (1.5) are oscillatory. In fact $\{y_n\}$ $=\{(-1)^n\}$ is one such solution of the equation (1.5)

REFERENCES

[1]. R. P. Agarwal: Difference equation and inequalities- theory, methods and Applications- 2nd edition.

[2]. R.P.Agarwal, Martin Bohner, Said R.Grace, Donal O'Regan: Discreteoscillation theory-CMIA Book Series, Volume 1, ISBN : 977-5945-19-4.

[3]. B.Selvaraj and I.Mohammed Ali Jaffer :Oscillation Behavior of Certain Third order Linear Difference Equations-FarEast Journal of Mathematical Sciences, Volume 40, Number 2, pp 169-178 (2010).

[4]. B.Selvaraj and I.Mohammed Ali Jaffer:Oscillatory Properties of Fourth OrderNeutral Delay ifference Equations-Journal of Computer and MathematicalSciences-An Iternational ResearchJournal,Vol. 1(3), 364-373 (2010).

[5]. B. Selvaraj and I. Mohammed AliJaffer: Oscillation Behavior of CertainThird order Non-linear DifferenceEquations-International Journal of Nonlinear Science (Accepted on September 6, (2010).

[6]. B.Selvaraj and I.Mohammed Ali Jaffer : Oscillation Theorems of Solutions ForCertain Third Order Functional Difference Equations With Delay- Bulletin of Pure and Applied Sciences (Accepted on October 20, (2010).

[7]. E.Thandapani and B.Selvaraj: Existence and Asymptotic Behavior of Nonoscillatory Solutions of Certain NonlinearDifference equation- Far East Journal of Mathematical Sciences 14(1), pp: 9-25 (2004).

[8]. E.Thandapani and B.Selvaraj:Oscillatory and Non-oscillatory Behavior of Fourth order Quasi-linear Difference equation -Far East Journal of Mathematical Sciences 17(3), 287-307 (2004).

[9]. E.Thandapani and B. Selvaraj: Oscillation of Fourth order Quasi-linear Difference equation-Fasci culi Mathematici Nr, 37, 109-119 (2007).