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A Comparison of Process Monitoring Schemes

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ABSTRACT: *In Statistical Process Control, control charts are be used to determine when a process may be out of control and in need of corrective action. In Demand Forecasting, tracking signals are used to monitor the accuracy of forecasts and to determine the presence of bias in a forecast model. This study compares the performance of an Individuals control chart and tracking signals in their ability to detect the presence of changes in a process mean (step shift) in a process consisting of independent observations.*
KEYWORDS*: Control Chart, Tracking Signals, Process Monitoring*

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I. INTRODUCTION

Control charting is arguably the most visible aspect of Statistical Process Control. The Shewhart Control Chart, also known as an Individuals Chart when applied to individual observations, the Exponentially Weighted Moving Average (EWMA) Control Chart and the Cumulative Sum (CUSUM) Control Chart have been used for a number of years to monitor manufacturing processes. Typically, in process control environments, monitoring schemes are compared based on their ability to detect step shifts in the level of a process.

In the forecasting and time series fields, tracking signals are used to monitor forecasting systems. A Smoothed Error (ETS) Tracking Signal and a Cumulative Sum (CUSUM) Tracking Signal are used to detect anomalies or bias in a forecast. Unusual behavior in the process should result in a large error that is reflected as a signal on a tracking signal.

Traditionally, monitoring tools have been compared based on Average Run Lengths (ARLs). The ARL is the expected number of observations required to detect an out-of-control situation. As an average measure that is inflated by long run lengths, the ARL is an inadequate measure of quick recovery, that is characterized by short run lengths. Hence the cumulative distribution function (CDF) of the run lengths is offered as an alternative criterion to the average run length (ARL) for the selection of an appropriate monitoring scheme. The CDF provides the cumulative probability of a signal occuring by the ith time period after a disturbance.

This paper compares the performance of an Individuals control chart, a Smoothed Error tracking signal, and a Cumulative Sum tracking signal in monitoring observations from a N (0, 1) process, in the presence of a changes in the process mean. The study shows that the ETS tracking signal offers the highest probability of detection of a small shift in a mean. The Individuals control offers the highest probability of early detection of a large shift in a process mean.

II. QUALITY CONTROL SCHEMES

In this study, the Individuals Control Chart, the Smoothed Error (ETS) and Cumulative Sum (CTS) tracking signals are applied to independent observations and their performances evaluated.

The Individuals Control Chart

The Individuals control chart applied to independent observations requires an estimate of the variance of observations. Defining the *i*th moving range to be

$$MR_i = |x_i - x_{i-1}|, \qquad i = 2, 3, \ldots, m \qquad (1)$$

and

$$\overline{MR} = \frac{1}{m-1}\sum_{i=2}^{m} MR_i , \qquad (2)$$

the control limits are

$$\overline{X} \pm C_1 MR/d_2 \qquad (3)$$

where the constant C_1 is set to achieve a desired in-control ARL. Montgomery (1991) has tabulated values for C_1 and d_2.

The Smoothed Error Tracking Signal
Trigg's (1964) Smoothed Error (ETS) tracking signal is given by

$$ETS_t = |E_t / MAD_t| \quad (4)$$
where
$$E_t = \alpha_1 e_t + (1-\alpha_1)E_{t-1}, \quad 0 \leq \alpha_1 \leq 1 \quad (5)$$
and
$$MAD_t = \alpha_2 |e_t| + (1-\alpha_2)MAD_{t-1}, \quad 0 \leq \alpha_2 \leq 1 . \quad (6)$$

Typically, $E_0 = 0$ and MAD_0 is set equal to its expected value which is approximately equal to $0.8\sigma_e$ (where σ_e is the standard deviation of the observations). A signal occurs if ETS_t exceeds a critical value K_1. Gardner (1983) suggests that the value of K_1 should be set to achieve a desired in-control ARL.

The Cumulative Sum Tracking Signal
Brown's (1959) Cumulative Sum (CTS) tracking signal is given by

$$CTS_t = |SUM_t / MAD_t| \quad (7)$$
where
$$SUM_t = e_t + SUM_{t-1} . \quad (8)$$

The value of MAD_0 is set equal to its expected value as with ETS_0. The value of SUM_0 is set equal to zero. A signal occurs if the value of CTS_t exceeds a critical value K_2. Gardner (1983) suggests that the value of K_2 should be set to achieve a desired in-control ARL.

Traditionally, the smoothing parameters in the numerator and denominator of the ETS have been set equal to each other, that is, $\alpha_1=\alpha_2$. More recently, McClain (1988) has suggested that the smoothing parameter in the MAD,α_2, be smaller than the parameter in the numerator, α_1, so that the variance of the observations may be stabilized.

The ARL is a criterion on which the relative performance of both tracking signals has been based. The use of the cumulative distribution functions (CDF) as an evaluation criterion is not new. Barnard (1959), Bissell (1968) and Gan (1991) recommend its use on independent observations. Referred to as a ‘response to a change in demand', McClain (1988) advocates its use for forecast-based schemes which incorporate tracking signals. The CDF measures the cumulative percentage of disturbances in a data stream that are detected early.

III. DESIGN OF THE SIMULATION STUDY

In this simulation study, three monitoring schemes were compared. They are the Individuals control chart, ETS tracking signal and CTS tracking signal. ARLs and CDFs are provided for each monitoring scheme for step shifts ranging from 0.0σ to 3.0σ in increments of 0.5 (recall $\sigma=1$).

The initial values of the smoothed error for the ETS (equation 4) and the sum of observations for the CTS (equation 7) are set to zero as suggested by Gardner (1985) and McClain (1988). The smoothing constants α_1 and α_2 were set to 0.10 as suggested by McKenzie (1978).

The simulation study was conducted as follows:

i) N (0,1) series are generated by the IMSL (1991, p.1350-1351) subroutine RNARM / DRNARM.

ii) the first fifty observations are used to allow for a burn-in period,

ii) fifty (50) preliminary sequences of observations are used to estimate the variance of the observations for a step increase of zero (the in-control state),

iv) tracking signals and control charts are constructed based on the estimates obtained in step (iv). The initial MAD values are set to $0.8\sigma_e$ (σ_e is the standard deviation of the observations) as suggested by Montgomery, Johnson and Gardiner (1990),

v) the monitoring schemes are applied to the observations,

vi) steps (i)-(v) are repeated 1000 times. For each monitoring scheme, the run length for each simulation iteration is recorded. These run lengths are used to obtain the ARLs and CDFs after a shift of size 0.0, 0.5σ, 1σ, 1.5σ, 2σ, 2.5σ and 3σ.

Table 1. Performance Comparisons with In-Control ARL=250, ϕ=0.0

Monitoring Scheme		ARL	% of signals given by the ith observation after the shift									
			1	2	3	4	5	6	7	8	9	10
Δ=0	Individuals	250	0.3	0.7	1.1	1.4	1.7	2.3	2.7	3.6	3.5	4.0
	CTS	252	0.2	0.3	0.3	0.3	0.4	0.4	0.5	0.7	0.8	0.8
	ETS	248	1.1	1.7	2.4	2.6	3.2	3.7	3.8	4.2	4.5	4.9
Δ=0.5	Individuals	111.0	0.8	1.8	2.3	3.2	4.0	5.0	5.7	6.2	7.0	7.8
	CTS	40.9	1.9	2.0	2.3	2.5	2.7	2.8	3.0	3.3	3.4	3.7
	ETS	26.2	1.3	1.7	3.0	3.6	5.2	7.1	9.1	11.2	13.5	14.9
Δ=1.0	Individuals	33.8	3.5	5.3	7.4	10.6	13.1	14.9	17.5	19.5	22.1	24.5
	CTS	26.8	0.7	0.7	0.8	0.9	1.0	1.2	1.5	1.9	2.2	2.5
	ETS	11.2	0.9	2.2	4.6	7.1	11.0	17.4	24.6	32.5	40.0	48.6
Δ=1.5	Individuals	11.5	8.5	17.8	25.9	33.5	37.6	42.6	47.1	51.0	54.6	59.7
	CTS	23.4	0.7	0.8	0.9	0.9	1.0	1.3	1.7	1.9	2.1	2.3
	ETS	7.2	1.7	4.2	9.9	17.7	11.7	41.9	55.9	68.7	78.9	88.2
Δ=2.0	Individuals	5.1	21.6	35.3	49.0	58.8	66.7	72.7	78.3	82.7	85.6	88.3
	CTS	22.4	0.8	0.8	0.8	0.9	1.0	1.0	1.2	1.2	1.3	1.5
	ETS	5.9	2.6	6.4	13.7	26.3	41.9	61.5	78.1	89.1	95.1	97.6
Δ=2.5	Individuals	2.9	35.9	58.6	72.4	80.6	87.2	92.4	94.4	96.2	97.3	98.2
	CTS	22.0	0.5	0.5	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.7
	ETS	5.0	2.5	9.0	22.8	40.4	61.8	80.0	91.3	96.8	99.6	99.9
Δ=3.0	Individuals	1.8	53.4	79.5	90.4	96.2	98.4	99.1	99.9	100	100	100
	CTS	21.9	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
	ETS	4.4	3.0	11.1	28.8	53.6	76.4	91.1	98.1	99.5	99.3	100

IV. SIMULATION RESULTS

Table I displays simulated ARLs and CDFs for the Individuals control chart, ETS and CTS tracking signals applied to observations from a N(0, 1) process after a shift size 0.0, 0.5σ, 1σ, 1.5σ, 2σ, 2.5σ and 3σ.
The results may be summarizes as follows:

1. For step shifts of 0.5σ and 1σ, the ETS has the shortest ARL (26.2, 11.2) and the highest probability of detection by the 10^{th} observation after the shift (14.9, 48.6).
2. For a step shifts of 2σ, 2.5σ and 3σ, the Individuals chart has the shortest ARL (5.1, 3.0, 1.8) and the highest probability of early detection by the first observation after the shift (21.6%, 35.8%, 53.4%).

V. CONCLUSIONS

This paper has compared quality control schemes for monitoring independent observations in the presence shifts in a process mean. For detecting small shifts in a process mean, the Smoothed Error tracking signal is recommended over the Individuals control chart and the CUSUM tracking signals as it offers the shortest average run length and the highest probability of detection of change in a process mean. For detecting larger shifts in a process mean, the Individuals control chart is recommended over the Smoothed Error and CUSUM tracking signals as it offers the shortest run length and the highest probability of early detection by the first observation after a shift in the process mean.

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