International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321- 4767 P-ISSN: 2321-4759
www.ijmsi.org Volume 8 Issue 7 || July, 2020 || PP 18-28

Multiple Mellin Transform of Product of I -Function of One Variable And I- Function of Several Variables.

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ABSTRACT:
In this paper a multiple Mellin Transform of product of I- function of one variable and I- function of several variables has been established. Certain special cases are also given.
KEY WORDS*: I- function, Mellin transform, Laplace transform Mellin-Barnes contour integral.*

Date of Submission: 29-08-2020 Date of Acceptance: 14-09-2020

I. INTRODUCTION:

The multiple Mellin transform of a function $f(x_1,.....x_r)$ is defined as

$$M[f(x_1,.....x_r); s_1,...., s_r] = \int_0^\infty \cdots \int_0^\infty x_1^{s_1-1} \cdots x_r^{s_r-1} f(x_1,\ldots,x_r)dx_1\ldots dx_r$$

(1.1)

The multiple Laplace transform of a function $f(x_1,.....x_r)$ is defined as

$$F(p_1,.....p_r) = \int_0^\infty \cdots \int_0^\infty e^{-p_1t_1} \cdots e^{-p_rt_r-1} f(x_1,\ldots,x_r)dx_1\ldots dx_r$$

(12)

The I- function will be defined and represented by Rathie [5] in following Mellin –Barnes type contour integral:

$$I_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_1,\alpha_1,A_j),\ldots\ldots\ldots,(a_{p_1},\alpha_p,A_p)\\(b_1,\beta_1\ ,B_1),\ldots\ldots\ldots,(b_q,\beta_q,B_q)\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_c \phi(s)z^s ds ,$$

(1.3)

where

$$\phi(s) = \frac{\prod_{j=1}^{m}\Gamma^{B_j}(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma^{B_j}\left(1-b_j+-\beta_j s\right)\prod_{j=n+1}^{p}\Gamma^{A_j}\left(aj-\alpha_j s\right)}$$

(1.4)

also m ,n, p, q are integers satisfying

$0 \le m \le q, 0 \le n \le p$, $z \ne 0$, an empty product is interpreted as unity.

$\alpha_j,(j=1,2,...,p)$, $\beta_j,(j=1,2,...,q)$, $A_j,(j=1,2,...,p)$, $B_j,(j=1,2,...,q)$ are positive numbers.

$a_j,(j=1,2,...,p)$, $b_j,(j=1,2,...,q)$ are complex numbers such that no singularity of $\Gamma^{B_j}(b_j-\beta_j s),(j=1,\ldots,m)$ coincides with any singularity of $\Gamma^{A_j}(1-a_j+\alpha_j s),(j=1,\ldots,n)$.

If $A_j = 1, (j = 1,2,...., n)$, $B_j = 1, (j = 1,2,...., m)$ in (1.4), it will be denoted by

$$\bar{I}^{m,n}_{p,q}\left[z \middle| \begin{matrix} (a_j, \alpha_j, 1)_{1,n}, (a_j, \alpha_j, A_j)_{n+1,p} \\ (b_j, \beta_j, 1)_{1,m}, (b_j, \beta_j, B_j)_{m+1,q} \end{matrix} \right] = \frac{1}{2\pi i}\int_c \bar{\phi}(s) z^s ds$$

(1.5)

where ,

$$\bar{\phi}(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q}\Gamma^{B_j}\left(1 - b_j + -\beta_j s\right)\prod_{j=n+1}^{p}\Gamma^{A_j}\left(aj - \alpha_j s\right)}$$

(1.6)

The I- function of r- variables defined and represented by Nambisan [8] et.al as:

$$\bar{I}^{0,n;(m_1,n_1);........;(m_r,n_r)}_{P,Q:(p_1,q_1);........;(p_r,q_r)}\left[\begin{matrix} z_1 \\ . \\ . \\ z_r \end{matrix} \middle| \begin{matrix} (a_j, \alpha_j^{(1)}, ..., \alpha_j^{(r)}, A_j)_{1,P}\, (c_j^{(1)}, \gamma_j^{(1)}, C_j^{(1)})_{1,p_1}, ..., (c_j^{(r)}, \gamma_j^{(r)}, C_j^{(r)})_{1,p_r} \\ \\ \\ (b_j, \beta_j^{(1)}, ..., \beta_j^{(r)}, B_j)_{1,Q} : (d_j^{(1)}, \delta_j^{(1)}, D_j^{(1)})_{1,q_1}; ...., (d_j^{(r)}, \delta_j^{(r)}, D_j^{(r)})_{1,q_r} \end{matrix} \right]$$

$$= \frac{1}{(2\pi i)^r}\int_{c_1}\cdots\int_{c_r} \phi(s_1, ..., s_r)\theta_1(s_1)\ldots\theta_r(s_r) z_1^{s_1}\ldots z_r^{s_r} ds_1\ldots ds_r$$

(1.7)

where

$$\phi(s_1 \ldots, s_r) = \frac{\prod_{j=1}^{n}\Gamma^{A_j}(1 - a_j + \sum_{i=1}^{r}\alpha_j^{(i)} s_i)}{\prod_{j=n+1}^{P}\Gamma^{A_j}(a_j - \sum_{j=1}^{r}\alpha_j^{(i)} s_i)\prod_{j=1}^{Q}\Gamma^{B_j}(1 - b_j + \sum_{i=1}^{r}\beta_j^{(i)} s_i)}, \quad \text{i=1,2,....,r}$$

(1.8)

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}(1 - c_j^{(i)} + \gamma_j^{(i)} s_i)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}(d_j^{(i)} - \delta_j^{(i)} s_i)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(k)} - \gamma_j^{(i)} s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1 - d_j^{(i)} + \delta_j^{(i)} s_i\right)}$$

i = 1,2,....,r

(1.9)

Also $z_i \neq 0$, i=1,2,....,r, an empty product is interpreted as unity.

The parameters $m_j, n_j, p_j, q_j, (j = 1,....., r), n, p, q$ are non –negative integers such that $0 \le n \le p, q \ge 0, 0 \le n_j \le p_j, 0 \le m_j \le q_j, (j = 1,2,...., r)$,not all zero simultaneously.

$\alpha_j^{(i)}, (j = 1,..., P, i = 1,...., r)$, $\beta_j^{(i)}, (j = 1,..., Q, i = 1,...., r)$, $\gamma_j^{(i)}, (j = 1,..., p_i, i = 1,...., r)$, $\delta_j^{(i)}, (j = 1,..., q_i, i = 1,...., r)$ are assumed to be positive quantities.

$a_j,(j=1,2,...,p)$, $b_j,(j=1,2,...,q)$, $c_j^{(i)},(j=1,\dots,p_i,i=1,\dots,r)$, $d_j^{(i)},(j=1,\dots,q_i,i=1,\dots,r)$ are complex numbers. All the singularities of $\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i),\ j=1,\dots,m_i$ lie to the right and $\Gamma^{C_j^{(i)}}(1-c_j^{(i)}+\gamma_j^{(i)}s_i),\ j=1,\dots,n_i$ are to the left of c_i

The I- function of r- variables is analytic by Braaksma, [1], if

$$\psi_k=\sum_{j=1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{p_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=1}^{q_k}D_j^{(k)}\delta_j^{(k)}\le 0,k=1,2,\dots,r \quad (1.10)$$

The integral (1.7) converges absolutely if

$\left|\arg z_k\right|<\frac{1}{2}\Delta_k\pi,k=1,\dots,r$, where

$$\Delta_k=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}>0.$$

and if

$$\left|\arg z_k\right|<\frac{1}{2}\Delta_k\pi,\Delta_k\ge 0,k=1,\dots,r,$$

then integral (2.1) converges absolutely under the following conditions.

i) $\psi_k=0,\Omega_k<-1$, where ψ_k given by (1.10) and for k= 1,2,…,r

$$\Omega_k=\sum_{j=1}^{p}[\tfrac{1}{2}-\mathrm{Re}(a_j)]A_j-\sum_{j=1}^{q}[\tfrac{1}{2}-\mathrm{Re}(b_j)]B_j+\sum_{j=1}^{p_k}[\tfrac{1}{2}-\mathrm{Re}(c_j^{(k)})]C_j^{(k)}-\sum_{j=1}^{q_k}[\tfrac{1}{2}-\mathrm{Re}(d_j^{(k)})]D_j^{(k)}.$$

$\psi_k\ne 0$, with $s_k=\sigma_k+it_k,(\sigma_k,t_k)$ are real numbers, (k=1, 2,…, r) are chosen that $\left|t_k\right|\to\infty$, $\Omega_k+\sigma_k\psi_k<-1$.

If

$C_j^{(i)}=1,(j=1,\dots,n_i),i=1,\dots,r)$, $D_j^{(i)}=1,(j=1,\dots,m_i),i=1,\dots,r)$ and n = 0, in (1.7), the corresponding function will be denoted by

$$\bar{I}_1[z_1\dots z_r]=I_{P,Q:(p_1,q_1);\dots\dots;(p_r,q_r)}^{0,0;(m_1,n_1);\dots\dots;(m_r,n_r)}\left[\begin{matrix}z_1\\ \cdot\\ \cdot\\ z_r\end{matrix}\left|\begin{matrix}(a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)},A_j)_{1,P}(c_j^{(1)},\gamma_j^{(1)},1)_{1,n_1}(c_j^{(1)},\gamma_j^{(1)},C_j^{(1)})_{n_1+1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)},1)_{1,n_r}(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{n_r+1,p_r}\\ \\ \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)},B_j)_{1,Q}:(d_j^{(1)},\delta_j^{(1)},1)_{1,m_1}(d_j^{(1)},\delta_j^{(1)},D_j^{(1)})_{m_1+1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)},1)_{1,m_r}(d_j^{(r)},\delta_j^{(r)},D_j^{(r)})_{m_r+1,q_r}\end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^r} \int_{c_1} \ldots \int_{c_r} \phi(s_1, \ldots, s_r) \bar{\theta}_1(s_1) \ldots \bar{\theta}_r(s_r) z_1^{s_1} \ldots z_r^{s_r} ds_1 \ldots ds_r$$

(1.11)

Where $$\phi(s_1 \ldots, s_r) = \frac{1}{\prod_{j=1}^{P} \Gamma^{A_j}(a_j - \sum_{j=1}^{r} \alpha_j^{(i)} s_i) \prod_{j=1}^{Q} \Gamma^{B_j}(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} s_i)}$$

(1.12)

$$\bar{\theta}_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma(1 - c_j^{(i)} + \gamma_j^{(i)} s_i) \prod_{j=1}^{m_i} \Gamma(d_j^{(i)} - \delta_j^{(i)} s_i)}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}\left(c_j^{(k)} - \gamma_j^{(i)} s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}\left(1 - d_j^{(i)} + \delta_j^{(i)} s_i\right)}$$

(1.13)

The integral (1.11) converges absolutely if $\left|\arg z_k\right| < \frac{1}{2} \Delta_k^{/} \pi, k = 1, \ldots, r$, where

$$\Delta_k^{/} = -\sum_{j=n+1}^{p} A_j \alpha_j^{(k)} - \sum_{j=1}^{q} B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} > 0 .$$

$$\left|\arg z_k\right| < \frac{1}{2} \Delta_k \pi, \Delta_k \geq 0, k = 1, \ldots, r ,$$

then integral (2.1) converges absolutely under the following conditions.

i) $\psi_k^{/} = 0, \Omega_k^{/} < -1$, where for k= 1,….,r

$$\psi_k^{/} = \sum_{j=1}^{p} A_j \alpha_j^{(k)} - \sum_{j=1}^{q} B_j \beta_j^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)} - \sum_{j=1}^{q_k} D_j^{(k)} \delta_j^{(k)} \leq 0,$$

and

$$\Omega_k^{/} = \sum_{j=1}^{p} [\tfrac{1}{2} - \mathrm{Re}(a_j)] A_j - \sum_{j=1}^{q} [\tfrac{1}{2} - \mathrm{Re}(b_j)] B_j + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=1}^{m_k} [\tfrac{1}{2} - \mathrm{Re}(d_j^{(k)})] - \sum_{j=m_k+1}^{q_k} [\tfrac{1}{2} - \mathrm{Re}(d_j^{(k)}),$$

$\psi_k^{/} \neq 0$, with $s_k = \sigma_k + i t_k, (\sigma_k, t_k)$ are real numbers, k=1,2,…,r are chosen that $\left|t_k\right| \to \infty$,

$$\Omega_k^{/} + \sigma_k \psi_k^{/} < -1$$

In (1.7) , put r =2, it reduces to the I- function of two variables given by Nambisan and Santhakumari.

Prathima and Nambisan [9]

$$\int_0^\infty \ldots \int_0^\infty x_1^{s_1 - 1} \ldots . x_r^{s_r - 1} \bar{I}\left[t_1 x_1^{\lambda_1}, \ldots t_r x_r^{\lambda_r}\right] dx_1 \ldots dx_r$$

$$= \frac{1}{\lambda_1 \ldots \lambda_r} \phi\left(\frac{-s_1}{\lambda_1}, \ldots \frac{-s_r}{\lambda_r}\right) \theta_1\left(\frac{-s_1}{\lambda_1}\right) \ldots . \theta_r\left(\frac{-s_r}{\lambda_r}\right) t_1^{\frac{-s_1}{\lambda_1}} \ldots . t_r^{\frac{-s_r}{\lambda_r}}$$

(1.14)

Where

$$\phi\left(\frac{-s_1}{\lambda_1},\ldots.-\frac{-s_r}{\lambda_r}\right)=\frac{1}{\prod_{j=1}^{p}\Gamma^{A_j}\left(a_j+\sum_{k=1}^{r}\alpha_j^{(k)}\frac{s_k}{\lambda_k}\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j-\sum_{k=1}^{r}\beta_j^{(k)}\frac{s_k}{\lambda_k}\right)},$$

(1.15)

$$\theta_k\left(\frac{-s_k}{\lambda_k}\right)=\frac{\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}-\gamma_j^{(k)}\frac{s_k}{\lambda_k})\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}+\delta_j^{(k)}\frac{s_k}{\lambda_k})}{\prod_{j=n_k+1}^{p_k}\Gamma^{C_j^{(k)}}\left(c_j^{(k)}+\gamma_j^{(k)}\frac{s_k}{\lambda_k}\right)\prod_{j=m_k+1}^{q_k}\Gamma^{D_j^{(k)}}\left(1-d_j^{(k)}-\delta_j^{(k)}\frac{s_k}{\lambda_k}\right)}$$

(1.16).

Provided,

$$-\lambda_i\min_{1\le m_i}\left(\mathrm{Re}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)<\mathrm{Re}(s_i)<\lambda_i\min_{1\le j\le n_i}\mathrm{Re}\left(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}\right),$$ I = 1,2,3,….,r.

$$\psi_i^{//}\le 0,\Delta_i^{//}\le 0,\ |\arg t_i|<\frac{1}{2}\Delta_i^{//}\pi .$$

Where, for k=1, 2,…, r

$$\psi_i^{//}=\sum_{j=1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}+\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=1}^{m_k}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)},$$

$$\Delta_i^{//}=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=m+1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}\delta_j^{(k)}+\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}$$

.

Prasanth and Nambisan [7]

If $F(p_1,\ldots.. p_r)$ be the Laplace transform and $G(s_1,\ldots.. s_r)$ be the Mellin transform of $f(t_1,\ldots..t_r)$, then

$$F(p_1,\ldots.. p_r)=\sum_{s_1=0}^{\infty}\ldots.\sum_{s_r=0}^{\infty}\frac{(-p_1)^{s_1}}{s_1!}\ldots.\frac{(-p_r)^{s_r}}{s_r!}G(s_1+1,\ldots,s_r+1)$$

(1.17)

II. MAIN RESULT:

$$\int_0^\infty\ldots..\int_0^\infty x_1^{\eta_1-1}\ldots\ldots x_r^{\eta_r-}I_{p,q}^{m,n}\left[yx_1^{\lambda_1}\ldots.x_r^{\lambda_r}\left|\begin{matrix}(g_j,G_j,1)_{1,n}(g_j,G_j,E_j)_{n+1,p}\\(h_j,H_j,1)_{1,m}(h_j,H_J,F_j)_{m+1,q}\end{matrix}\right.\right]\times$$

$$\bar{I}_{1P,Q:(p_1,q_1);\ldots\ldots..;(p_r,q_r)}^{0,0;(m_1,n_1);\ldots\ldots..;(m_r,n_r)}$$

$$\left[\begin{matrix}z_1x_1^{\mu_1}\\ .\\ .\\ z_rx_r^{\mu_r}\end{matrix}\left|\begin{matrix}(a_j,\alpha_j^{(1)},\ldots,\alpha_j^{(r)},A_j)_{1,P}(c_j^{(1)},\gamma_j^{(1)},1)_{1,n_1}(c_j^{(1)},\gamma_j^{(1)},C_j^{(1)})_{n_1+1,,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)},1)_{1,n_r}(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{n_r+1,,p_r}\\ \\ \\(b_j,\beta_j^{(1)},\ldots.,\beta_j^{(r)},B_j)_{1,Q}:(d_j^{(1)},\delta_j^{(1)},1)_{1,m_1}(d_j^{(1)},\delta_j^{(1)},D_j^{(1)})_{m_1+1,,q_1};..;(d_j^{(r)},\delta_j^{(r)},1)_{1,m_r}(d_j^{(r)},\delta_j^{(r)},D_j^{(r)})_{m_r+1,,q_r}\end{matrix}\right.\right]$$

$$\times dx_1......dx_r \; .$$

$$= \frac{1}{\mu_1 \ldots \mu_r} z_1^{\frac{-\eta_1}{\mu_1}} \ldots z_r^{\frac{-\eta_r}{\mu_r}}$$

$$I^{m+n_1+.......+n_r, n+m_1+........+m_r}_{p+Q+q_1+...+q_r, P+q+p_1+..+p_r} \left[y z_1^{\frac{-\lambda_1}{\mu_1}} \ldots z_r^{\frac{-\lambda_r}{\mu_r}} \middle| \begin{matrix} M \\ N \end{matrix} \right], \qquad (2.1)$$

Where,

$$M \equiv (g_j, G_j, 1)_{1,n} (1-d_j^{(1)} - \frac{\eta_1}{\mu_1}\delta_j^{(1)}, \frac{\lambda_1}{\mu_1}\delta_j^{(1)}, 1)_{1,m_1}, ..., (1-d_j^{(r)} - \frac{\eta_r}{\mu_r}\delta_j^{(r)}, \frac{\lambda_r}{\mu_r}\delta_j^{(r)}, 1)_{1,m_r}$$

$$(g_j, G_j, E_j)_{n+1,p} (1-d_j^{(1)} - \frac{\eta_1}{\mu_1}\delta_j^{(1)}, \frac{\lambda_1}{\mu_1}\delta_j^{(1)}, E_j)_{m_1+1,q_1}, ..., (1-d_j^{(r)} - \frac{\eta_r}{\mu_r}\delta_j^{(r)}, \frac{\lambda_r}{\mu_r}\delta_j^{(r)}, E_j)_{m_r+1,q_r}$$

$$(1-b_j - \frac{\eta_1}{\mu_1}\beta_j^{(1)} - ....- \frac{\eta_r}{\mu_r}\beta_j^{(r)}, \frac{\lambda_1}{\mu_1}\beta_j^{(1)} +, ..., + \frac{\lambda_r}{\mu_r}\beta_j^{(r)}, B_j)_{1,Q} \; .$$

$$N \equiv (h_j, H_j, 1)_{1,m} (1-c_j^{(1)} - \frac{\eta_1}{\mu_1}\gamma_j^{(1)}, \frac{\lambda_1}{\mu_1}\gamma_j^{(1)}, 1)_{1,n_1}, ..., (1-c_j^{(r)} - \frac{\eta_r}{\mu_r}\gamma_j^{(r)}, \frac{\lambda_r}{\mu_r}\gamma_j^{(r)}, 1)_{1,n_r}$$

$$(h_j, H_j, F_j)_{m+1,q} (1-c_j^{(1)} - \frac{\eta_1}{\mu_1}\gamma_j^{(1)}, \frac{\lambda_1}{\mu_1}\gamma_j^{(1)}, F_j)_{n_1+1,p_1}, ..., (1-c_j^{(r)} - \frac{\eta_r}{\mu_r}\gamma_j^{(r)}, \frac{\lambda_r}{\mu_r}\gamma_j^{(r)}, F_j)_{n_r+1,p_r}$$

$$(1-a_j - \frac{\eta_1}{\mu_1}\alpha_j^{(1)} - ....-, \frac{\eta_r}{\mu_r}\alpha_j^{(r)}, \frac{\lambda_1}{\mu_1}\alpha_j^{(1)} +, ..., + \frac{\lambda_r}{\mu_r}\alpha_j^{(r)}, A_j)_{1,P} \; .$$

Provided,

i) $\lambda_i, \mu_i > 0, (1 \le i \le r)$

ii)

$$-\lambda_i \min_{1\le j\le m} \mathrm{Re}(\frac{h_j}{H_j}) - \mu_i \min_{1\le j\le m_i} \mathrm{Re}(\frac{d_j^{(i)}}{\delta_j^{(i)}}) < \mathrm{Re}(\eta_i) < \lambda_i \min_{1\le j\le n} \mathrm{Re}(\frac{1-g_j}{G_j}) + \mu_i \min_{1\le j\le n_i} (\frac{1-c_j^{(i)}}{\gamma_j^{(i)}}) \; .$$

iii) $|\arg y| < \frac{1}{2}\Delta\pi$, $|\arg z_i| < \frac{1}{2}\Delta_i\pi$, $\Delta > 0,$ $\Delta_i > 0, \Delta_i > 0, A_i \le 0, \delta \le 0, \Delta_i > 0$

where ,

$$\delta = \sum_{j=m+1}^{q} G_j - \sum_{j=1}^{p} H_j$$

$$\Delta = \sum_{j=1}^{m} H_j - \sum_{j=m+1}^{q} H_j + \sum_{j=1}^{n} G_j - \sum_{j=n+1}^{p} G_j$$

$$\Delta_i = \sum_{j=1}^{p} \alpha_j^{(i)} + \sum_{j=1}^{n_i} c_j^{(i)} - \sum_{j=n_i+1}^{p_i} c_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{m_i} d_j^{(i)} - \sum_{j=m_i+1}^{q_i} d_j^{(i)}, i = 1, \ldots, r .$$

$$A_i = \sum_{j=1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_J^{(i)} + \sum_{j=1}^{p_i} c_j^{(i)} - \sum_{j=1}^{q_i} d_j^{(i)}, i = 1,2,\ldots, r$$

PROOF:

To prove (2.1) substitute Mellin- Barnes contour integral for $I\left[yx_1^{\lambda_1} \ldots\ldots x_r^{\lambda_r}\right]$ on left side with the help of (1.5), interchange the order of **s** and $(x_1, \ldots, x_r)$ integrals , evaluate the $(x_1, \ldots, x_r)$ integral using (1.14), then interpret the resulting integral with the help of (1.5).

SPECIAL CASES:

When r =2,

$$\int_0^\infty \int_0^\infty x_1^{\eta_1 - 1} x_2^{\eta_2 - 1} I_{p,q}^{m,n}\left[yx_1^{\lambda_1} x_2^{\lambda_2} \middle| \begin{matrix} (g_j, G_j, 1)_{1,n} (g_j, G_j, E_j)_{n+1,p} \\ (h_j, H_j, 1)_{1,m} (h_j, H_J, F_j)_{m+1,q} \end{matrix}\right] \times$$

$$\bar{I}_{1P,Q:(p_1,q_1);(p_2,q_2)}^{0,0;(m_1,n_1);(m_2,n_2)}$$

$$\left[\begin{matrix} z_1 x_1^{\mu_1} \\ \\ z_2 x_2^{\mu_2} \end{matrix} \middle| \begin{matrix} (a_j, \alpha_j, A_j, \xi_j)_{1,P} (c_j^{(1)}, \gamma_j^{(1)}, 1)_{1,n_1} (c_j^{(1)}, \gamma_j^{(1)}, C_j)_{n_1+1,p_1}; (c_j^{(2)}, \gamma_j^{(2)}, 1)_{1,n_{2r}} (c_j^{(2)}, \gamma_j^{(r)}, C_j^{/})_{n_2+1,p_2} \\ \\ (b_j, \beta_j, B_j, k_j)_{1,Q} : (d_j^{(1)}, \delta_j^{(1)}, 1)_{1,m_1} (d_j^{(1)}, \delta_j^{(1)}, D_j)_{m_1+1,q_1} (d_j^{(2)}, \delta_j^{(2)}, 1)_{1,m_2} (d_j^{(2)}, \delta_j^{(2)}, D_j^{/})_{m_2+q_2} \end{matrix}\right]$$

$$\times dx_1.dx_2$$

$$= \frac{1}{\mu_1 \mu_2} z_1^{\frac{-\eta_1}{\mu_1}} z_2^{\frac{-\eta_2}{\mu_2}} I_{p+Q+q_1+q_2, P+q+p_1+p_2}^{m+n_1+n_2, n+m_1+m_2}\left[yz_1^{\frac{-\lambda_1}{\mu_1}} z_2^{\frac{-\lambda_2}{\mu_2}} \middle| \begin{matrix} M \\ N \end{matrix}\right] \tag{2.2}$$

Where

$$M \equiv (g_j, G_j, 1)_{1,n} (1 - d_j^{(1)} - \frac{\eta_1}{\mu_1}\delta_j^{(1)}, \frac{\lambda_1}{\mu_1}\delta_j^{(1)}, 1)_{1,m_1}, (1 - d_j^{(2)} - \frac{\eta_2}{\mu_2}\delta_j^{(2)}, \frac{\lambda_2}{\mu_2}\delta_j^{(2)}, 1)_{1,m_2}$$

$$(g_j, G_j, E_j)_{n+1,p} (1 - d_j^{(1)} - \frac{\eta_1}{\mu_1}\delta_j^{(1)}, \frac{\lambda_1}{\mu_1}\delta_j^{(1)}, E_j)_{m_1+1,q_1}, (1 - d_j^{(2)} - \frac{\eta_2}{\mu_2}\delta_j^{(2)}, \frac{\lambda_2}{\mu_2}\delta_j^{(2)}, E_j)_{m_2+1,q_2}$$

$$(1 - b_j - \frac{\eta_1}{\mu_1}\beta_j - \frac{\eta_2}{\mu_2}B_j, \frac{\lambda_1}{\mu_1}\beta_j + \frac{\lambda_2}{\mu_2}B_j, \xi_j)_{1,Q} .$$

$$N \equiv (h_j, H_j, 1)_{1,m} (1-c_j^{(1)} - \frac{\eta_1}{\mu_1}\gamma_j^{(1)}, \frac{\lambda_1}{\mu_1}\gamma_j^{(1)}, 1)_{1,n_1}, (1-c_j^{(2)} - \frac{\eta_2}{\mu_2}\gamma_j^{(2)}, \frac{\lambda_2}{\mu_2}\gamma_j^{(2)}, 1)_{1,n_2}$$

$$(h_j, H_j, F_j)_{n+1,p} (1-c_j^{(1)} - \frac{\eta_1}{\mu_1}\gamma_j^{(1)}, \frac{\lambda_1}{\mu_1}\gamma_j^{(1)}, F_j)_{m_1+1,q_1}, (1-c_j^{(2)} - \frac{\eta_2}{\mu_2}\gamma_j^{(2)}, \frac{\lambda_2}{\mu_2}\gamma_j^{(2)}, F_j)_{m_2+1,q_2}$$

$$(1-a_j - \frac{\eta_1}{\mu_1}\alpha_j - \frac{\eta_2}{\mu_2}A_j, \frac{\lambda_1}{\mu_1}\alpha_j + \frac{\lambda_2}{\mu_2}A_j, C_j)_{1,P}.$$

Provided,

i) $\lambda_i, \mu_i > 0, (i = 1,2)$

ii)

$$-\lambda_i \min_{1\le j\le m} \mathrm{Re}(\frac{h_j}{H_j}) - \mu_i \min_{1\le j\le m_1} \mathrm{Re}(\frac{d_j^{(i)}}{\delta_j^{(i)}}) < \mathrm{Re}(\eta_i) < \lambda_i \min_{1\le j\le n} \mathrm{Re}(\frac{1-g_j}{G_j}) + \mu_i \min_{1\le j\le n_i}(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}})$$

.

iii) $|\arg y| < \frac{1}{2}\Delta\pi$, $|\arg z_i| < \frac{1}{2}\Delta_i\pi$, $\Delta > 0, \quad \Delta_i > 0,$ (i = 1,2), $U_1 \le 0, V_1 \le 0, \delta \le 0.$

where

$$\delta = \sum_{j=m+1}^{q} G_j - \sum_{j=1}^{p} H_j$$

$$\Delta = \sum_{j=1}^{m} H_j - \sum_{j=m+1}^{q} H_j + \sum_{j=1}^{n} G_j - \sum_{j=n+1}^{p} G_j$$

$$\Delta_1 = -\sum_{j=1}^{p} \alpha_j - \sum_{j=1}^{Q} \beta_J + \sum_{j=1}^{m_1} \delta_j^{(1)} - \sum_{j=m_1+1}^{q_1} \delta_j^{(1)} + \sum_{j=1}^{m_i} \gamma_j^{(1)} - \sum_{j=n_1+1}^{p_1} \gamma_j^{(1)}.$$

$$\Delta_2 = -\sum_{j=1}^{p} A_j - \sum_{j=1}^{Q} B_j + \sum_{j=1}^{m_2} \delta_j^{(2)} - \sum_{j=m_2+1}^{q_1} \delta_j^{(2)} + \sum_{j=1}^{n_2} \gamma_j^{(2)} - \sum_{j=n_2+1}^{p_2} \gamma_j^{(2)}.$$

$$U_1 = \sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{p_1} \gamma_j^{(1)} - \sum_{j=1}^{q} \beta_j - \sum_{j=n+1}^{q_1} \delta_j, \quad V_1 = \sum_{j=1}^{p} A_j + \sum_{j=1}^{p_2} \gamma_j^{(2)} - \sum_{j=1}^{q} B_j - \sum_{j=n+1}^{q_2} \delta_j^{(2)}.$$

Put

$E_j (j = 1,..., p)$, $F_j (j = m+1,..., q)$, $\xi_j (j = 1,..., P)$, $K_j (j = 1,..., Q)$, $C_j (j = n_1 + 1,..., p_1)$, $C_j^{/} (j = n_2 + 1,\ldots,p_2)$, $D_j (j = m_1 + 1,\ldots,q_1)$, $D_j^{/} (j = m_2 + 1,\ldots,q_2)$ are equal to unity and $\mu_1 = \mu_2 = 1$ in (2.2) , it reduces to the result given by Srivastava, Gupta, and Goyal[7,p,150].

APPLICATION:

$$\int_0^{\infty} \ldots \int_0^{\infty} e^{-p_1 x_1} \ldots e^{-p_r x_r} x_1^{\eta_1 - 1} \ldots\ldots x_r^{\eta_r -} I_{p,q}^{m,n} \left[y x_1^{\lambda_1} \ldots x_r^{\lambda_r} \middle| \begin{matrix} (g_j, G_j, 1)_{1,n} (g_j, G_j, E_j)_{n+1,p} \\ (h_j, H_j, 1)_{1,m} (h_j, H_J, F_j)_{m+1,q} \end{matrix} \right] \times$$

$$\bar{I}_{1P,Q:(p_1,q_1);\ldots\ldots\ldots;(p_r,q_r)}^{0,0;(m_1,n_1);\ldots\ldots\ldots;(m_r,n_r)}$$

$$\left[\begin{array}{l|l} z_1 x_1^{\mu_1} & (a_j,\alpha_j^{(1)},\ldots,\alpha_j^{(r)},A_j)_{1,P}\,(c_j^{(1)},\gamma_j^{(1)},1)_{1,n_1}\,(c_j^{(1)},\gamma_j^{(1)},C_j^{(1)})_{n_1+1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)},1)_{1,n_r}\,(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{n_r+p_r} \\ . & \\ . & \\ z_r x_r^{\mu_r} & (b_j,\beta_j^{(1)},\ldots,\beta_j^{(r)},B_j)_{1,Q} : (d_j^{(1)},\delta_j^{(1)},1)_{1,m_1}\,(d_j^{(1)},\delta_j^{(1)},D_j^{(1)})_{m_1+1};\ldots;(d_j^{(r)},\delta_j^{(r)},1)_{1,m_r}\,(d_j^{(r)},\delta_j^{(r)},D_j^{(r)})_{m_r+q_r} \end{array}\right]$$

$$\times dx_1 \ldots\ldots dx_r$$

$$= \frac{1}{\mu_1\ldots\mu_r} z_1^{-(\frac{\eta_1-1}{\mu_1})} \ldots z_r^{-(\frac{\eta_r-1}{\mu_r})} \sum_{s_1=0}^{\infty}\cdots\sum_{s_r=0}^{\infty} \frac{\left(z_1^{\frac{-p_1}{1/\mu_1}}\right)^{s_1}}{s_1!} \cdots\cdots \frac{\left(z_r^{\frac{-p_r}{1/\mu_r}}\right)^{s_r}}{s_r!}$$

$$I^{m+n_1+\ldots\ldots+n_r,\,n+m_1+\ldots\ldots+m_r}_{p+Q+q_1+\ldots+q_r,\,P+q+p_1+\ldots+p_r}\left[y z_1^{\frac{-\lambda_1}{\mu_1}} \ldots z_r^{\frac{-\lambda_r}{\mu_r}} \left| \begin{matrix} M \\ N \end{matrix} \right.\right],$$

(2.3)

Where,

$$M \equiv (g_j,G_j,1)_{1,n}(1-d_j^{(1)}-\frac{(s_1+\eta_1-1)}{\mu_1}\delta_j^{(1)},\frac{\lambda_1}{\mu_1}\delta_j^{(1)},1)_{1,m_1},\ldots,(1-d_j^{(r)}-\frac{(s_r+\eta_r-1)}{\mu_r}\delta_j^{(r)},\frac{\lambda_r}{\mu_r}\delta_j^{(r)},1)_{1,m_r}$$

$$(g_j,G_j,E_j)_{n+1,p}(1-d_j^{(1)}-\frac{(s_1+\eta_1-1)}{\mu_1}\delta_j^{(1)},\frac{\lambda_1}{\mu_1}\delta_j^{(1)},E_j)_{m_1+1,q_1},\ldots,(1-d_j^{(r)}-\frac{(s_r+\eta_r-1)}{\mu_r}\delta_j^{(r)},\frac{\lambda_r}{\mu_r}\delta_j^{(r)},E_j)_{m_r+1,q_r}$$

$$(1-b_j-\frac{(s_1+\eta_1-1)}{\mu_1}\beta_j^{(1)}-\ldots-\frac{(s_r+\eta_r-1)}{\mu_r}\beta_j^{(r)},\frac{\lambda_1}{\mu_1}\beta_j^{(1)}+,\ldots,+\frac{\lambda_r}{\mu_r}\beta_j^{(r)},B_j)_{1,Q}$$

.

$$N \equiv (h_j,H_j,1)_{1,m}(1-c_j^{(1)}-\frac{(s_1+\eta_1-1)}{\mu_1}\gamma_j^{(1)},\frac{\lambda_1}{\mu_1}\gamma_j^{(1)},1)_{1,n_1},\ldots,(1-c_j^{(r)}-\frac{(s_r+\eta_r-1)}{\mu_r}\gamma_j^{(r)},\frac{\lambda_r}{\mu_r}\gamma_j^{(r)},1)_{1,n_r}$$

$$(h_j,H_j,F_j)_{m+1,q}(1-c_j^{(1)}-\frac{(s_1+\eta_1-1)}{\mu_1}\gamma_j^{(1)},\frac{\lambda_1}{\mu_1}\gamma_j^{(1)},F_j)_{n_1+1,p_1},\ldots,(1-c_j^{(r)}-\frac{(s_r+\eta_r-1)}{\mu_r}\gamma_j^{(r)},\frac{\lambda_r}{\mu_r}\gamma_j^{(r)},F_j)_{n_r+1,p_r}$$

$$(1-a_j-\frac{(s_1+\eta_1-1)}{\mu_1}\alpha_j^{(1)}-\ldots-,\frac{(s_r+\eta_r-1)}{\mu_r}\alpha_j^{(r)},\frac{\lambda_1}{\mu_1}\alpha_j^{(1)}+,\ldots,+\frac{\lambda_r}{\mu_r}\alpha_j^{(r)},A_j)_{1,P}$$

.r

Provided,

$\lambda_i,\mu_i > 0,(1 \le i \le r)$

i)

ii)

$$-\lambda_i \min_{1\le j\le m} \mathrm{Re}(\frac{h_j}{H_j}) - \mu_i \min_{1\le j\le m_i} \mathrm{Re}(\frac{d_j^{(i)}}{\delta_j^{(i)}}) < \mathrm{Re}(\eta_i) < \lambda_i \min_{1\le j\le n} \mathrm{Re}(\frac{1-g_j}{G_j}) + \mu_i \min_{1\le j\le n_i}(\frac{1-c_j^{(i)}}{\gamma_j^{(i)}})$$

.

iii) $\left|\arg y\right| < \frac{1}{2}\Delta\pi$, $\left|\arg z_i\right| < \frac{1}{2}\Delta_i\pi$, $\Delta > 0, \ \Delta_i > 0, \Delta_i > 0, A_i \le 0, \delta \le 0, \Delta_i > 0$

where $\delta = \sum_{j=m+1}^{q} G_j - \sum_{j=1}^{p} H_j$

$$\Delta = \sum_{j=1}^{m} H_j - \sum_{j=m+1}^{q} H_j + \sum_{j=1}^{n} G_j - \sum_{j=n+1}^{p} G_j$$

$$\Delta_i = \sum_{j=1}^{p} \alpha_j^{(i)} + \sum_{j=1}^{n_i} c_j^{(i)} - \sum_{j=1}^{p_i} c_j^{(i)} - \sum_{j=1}^{Q} \beta_J^{(i)} + \sum_{j=1}^{m_i} d_j^{(i)} - \sum_{j=m_i+1}^{q_i} d_j^{(i)}$$. i = 1,2,....,r

$$A_i = \sum_{j=1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_J^{(i)} + \sum_{j=1}^{p_i} c_j^{(i)} - \sum_{j=1}^{q_i} d_j^{(i)}, i = 1,2,\ldots,r$$

PROOF:

In (1.17), put $f(t_1,\ldots,t_r) = x_1^{\eta_1-1}\ldots.x_r^{\eta_r-1}\, I_{p,q}^{m,n}\left[yx_1^{\lambda_1}\ldots x_r^{\lambda_r} \middle| \begin{matrix} (g_j,G_j,1)_{1,n}(g_j,G_j,E_j)_{n+1,p} \\ (h_j,H_j,1)_{1,m}(h_j,H_J,F_j)_{m+1,q} \end{matrix} \right] \times$

$$\bar{I}_{1P,Q:(p_1,q_1);\ldots\ldots;(p_r,q_r)}^{0,0;(m_1,n_1);\ldots\ldots;(m_r,n_r)}$$

$$\left[\begin{matrix} z_1 x_1^{\mu_1} \\ . \\ . \\ z_r x_r^{\mu_r} \end{matrix} \middle| \begin{matrix} (a_j,\alpha_j^{(1)},\ldots,\alpha_j^{(r)},A_j)_{1,P}(c_j^{(1)},\gamma_j^{(1)},1)_{1,n_1}(c_j^{(1)},\gamma_j^{(1)},C_j^{(1)})_{n_1+1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)},1)_{1,n_r}(c_j^{(r)},\gamma_j^{(r)},C_j^{(r)})_{n_r+p_r} \\ \\ \\ (b_j,\beta_j^{(1)},\ldots.,\beta_j^{(r)},B_j)_{1,Q}:(d_j^{(1)},\delta_j^{(1)},1)_{1,m_1}(d_j^{(1)},\delta_j^{(1)},D_j^{(1)})_{m_1+1};..;(d_j^{(r)},\delta_j^{(r)},1)_{1,m_r}(d_j^{(r)},\delta_j^{(r)},D_j^{(r)})_{m_r+q_r} \end{matrix} \right]$$

and use (2.1), to get:

$$G(s_1,\ldots.,s_r) = M(f(t_1,\ldots,t_r)$$

$$= \frac{1}{\mu_1\ldots\mu_r} z_1^{\frac{-(s_1+\eta_1)}{\mu_1}} \ldots z_r^{\frac{-(s_r+\eta_r)}{\mu_r}}\, I_{p+Q+q_1+\ldots+q_r,P+q+p_1+..+p_r}^{m+n_1+\ldots\ldots+n_r,n+m_1+\ldots\ldots+m_r}\left[yz_1^{\frac{-\lambda_1}{\mu_1}}\ldots z_r^{\frac{-\lambda_r}{\mu_r}} \middle| \begin{matrix} M \\ N \end{matrix} \right],$$

Where,

$$M \equiv (g_j,G_j,1)_{1,n}(1-d_j^{(1)} - \frac{s_1+\eta_1}{\mu_1}\delta_j^{(1)}, \frac{\lambda_1}{\mu_1}\delta_j^{(1)},1)_{1,m_1},\ldots.,(1-d_j^{(r)} - \frac{s_r+\eta_r}{\mu_r}\delta_j^{(r)}, \frac{\lambda_r}{\mu_r}\delta_j^{(r)},1)_{1,m_r}$$

$$(g_j,G_j,E_j)_{n+1,p}(1-d_j^{(1)} - \frac{s_1+\eta_1}{\mu_1}\delta_j^{(1)}, \frac{\lambda_1}{\mu_1}\delta_j^{(1)},E_j)_{m_1+1,q_1},\ldots.,(1-d_j^{(r)} - \frac{s_r+\eta_r}{\mu_r}\delta_j^{(r)}, \frac{\lambda_r}{\mu_r}\delta_j^{(r)},E_j)_{m_r+1,q_r}$$

$$(1-b_j-\frac{s_1+\eta_1}{\mu_1}\beta_j^{(1)}-\ldots-\frac{s_r+\eta_r}{\mu_r}\beta_j^{(r)},\frac{\lambda_1}{\mu_1}\beta_j^{(1)}+,\ldots,+\frac{\lambda_r}{\mu_r}\beta_j^{(r)},B_j)_{1,Q}$$

.

$$N\equiv(h_j,H_j,1)_{1,m}(1-c_j^{(1)}-\frac{s_1+\eta_1}{\mu_1}\gamma_j^{(1)},\frac{\lambda_1}{\mu_1}\gamma_j^{(1)},1)_{1,n_1},\ldots,(1-c_j^{(r)}-\frac{s_r+\eta_r}{\mu_r}\gamma_j^{(r)},\frac{\lambda_r}{\mu_r}\gamma_j^{(r)},1)_{1,n_r}$$

$$(h_j,H_j,F_j)_{m+1,q}(1-c_j^{(1)}-\frac{s_1+\eta_1}{\mu_1}\gamma_j^{(1)},\frac{\lambda_1}{\mu_1}\gamma_j^{(1)},F_j)_{n_1+1,p_1},\ldots,(1-c_j^{(r)}-\frac{s_r+\eta_r}{\mu_r}\gamma_j^{(r)},\frac{\lambda_r}{\mu_r}\gamma_j^{(r)},F_j)_{n_r+1,p_r}$$

$$(1-a_j-\frac{s_1+\eta_1}{\mu_1}\alpha_j^{(1)}-\ldots-,\frac{s_r+\eta_r}{\mu_r}\alpha_j^{(r)},\frac{\lambda_1}{\mu_1}\alpha_j^{(1)}+,\ldots,+\frac{\lambda_r}{\mu_r}\alpha_j^{(r)},A_j)_{1,P}$$

.

Hence $F(p_1,\ldots,p_r)$ = the right side of (2.3).

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