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First and Second Acceleration Poles in Unitary Geometry Homothetic Motions

Hasan Es
Gazi University, Faculty of Education, Department of Mathematics, Ankara-Turkey
E-mail: hasanes@gazi.edu.tr

ABSTRACT: *In this paper, using matrix methods, we obtained rotation pole in one-parameter homothetic motions and pole orbits, accelerations and combinations of accelerations, first and second in acceleration poles. Moreover, some new theorems are given.*
KEYWORDS: *One parameter motions, pole point, acceleration pole.are called absolute acceleration, sliding acceleration, relative acceleration and Coriolis accelerations, respectively. The solution of the equation.*
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I. INTRODUCTION

A general planar motion as given by

(1.1) $$y_1 = x\cos\theta - y\sin\theta + a$$
$$y_2 = x\sin\theta + y\cos\theta + b$$

If θ, a and b are given by the functions of time parameter t, then this motions is called as one parameter motion. One parameter planar motion given by (1.1) can be written in the form

$$\begin{pmatrix} Y \\ 1 \end{pmatrix} = \begin{pmatrix} A & C \\ 0 & 1 \end{pmatrix}\begin{pmatrix} X \\ 1 \end{pmatrix}$$

or

(1.2) $$Y = AX + C, \quad Y = [y_1 y_2]^T, X = [xy]^T, C = [a\ b]^T$$

where $A \in SO(2)$, and Yand Xare the position vectors of the same point B, respectively, for the fixed and moving systems, and C is the translation vector. By taking the derivates with respect to t in (1.2), we get

(1.3) $$\dot{Y} = \dot{A}X + A\dot{X} + \dot{C}$$

(1.4) $$V_a = V_f + V_r$$

where the velocites $V_a = \dot{Y}, V_f = \dot{A}X + \dot{C}, \quad V_r = A\dot{X}$are called absolute, sliding, and relative velocites of the points B, respectively. the solution of the equation $V_f = 0$ gives us the pole points on the moving plane. The locus of these points is called the moving pole curve, and correspondingly the locus of pole points on the fixed plane is called the fixed pole curve. by taking the derivates with respect to t in (1.3), we get

(1.5) $$\ddot{Y} = \ddot{A}X + 2\dot{A}\dot{X} + A\ddot{X} + \ddot{C}$$

(1.6) $$b_a = b_r + b_c + b_f$$

where the velocites

(1.7) $$b_a = \ddot{Y},$$

(1.8) $$b_f = \ddot{A}X + \ddot{C},$$

(1.9) $$b_r = A\ddot{X},$$

(1.10) $$b_c = 2\dot{A}\dot{X},$$

are called absolute acceleration, sliding acceleration, relative acceleration and Coriolis accelerations, respectively. The solution of the equation

(1.11) $$\ddot{A}X + \ddot{C} = 0$$

gives the acceleration pole of the motion.

II. HOMOTHETIC MOTION IN EUCLIDEAN PLANE

Definition 2.1. The transformation given by the matrix

$$F = \begin{pmatrix} hA & C \\ 0 & 1 \end{pmatrix}$$

is called Homothetic motion in E^4. Here $h = hI_4$ is a scalar matrix, $A \in SO(4)$ and $C = C \in IR_1^4$ [1].

Definition 2.2. Let $J \subset IR$ be an open interval let $O \in J$. The transformation $F(t) = E^4 \to E^4$ given by

$$F(t) = \begin{pmatrix} h(t)A(t) & C(t) \\ 0 & 1 \end{pmatrix}$$

is called one-parameter homothetic motion in E^4,where the function $h: J \to IR$ the matrix $A \in SO(4)$ and the 4×1 type matrix C are differentiable with respect to [1]. Since h is scalar we have $B^{-1} = h^{-1}A^{-1} = \frac{1}{h}A^T$ for $X \epsilon E^4$, the geometric plane of the points is a curve in E^4 [3]. We will denote this curve by

(2.1) $$Y(t) = B(t)X(t) + C(t)$$

differentiating with respect totwe obtain [3];

(2.2) $$\frac{dY}{dt} = \frac{dB}{dt}X + B\frac{dX}{dt} + \frac{dC}{dt}$$

Definition 2.3. Equation of the general motion in E^4

(2.3) $$Y(t) = B(t)X(t) + C(t)$$

where $A = A(t) \in SO(4)$ and $C = C(t) \in IR_1^4$ [1]. Differentiating this equation with respect to t we have

(2.4) $$\frac{dY}{dt} = \frac{dB}{dt}X + B\frac{dX}{dt} + \frac{dC}{dt}$$

Here

$$V_a = \frac{dY}{dt}, \ V_r = B\frac{dX}{dt} \ \text{ and } \ V_f = \frac{dB}{dt}X + \frac{dC}{dt}$$

and are called absolute velocity, relative velocity and sliding velocity of the motion, respectively [1]. We deonte motions in E^4 by $E/_{E'}$ where $E'$is fixed plane and E is the moving plane with respect to E'. If te matrix $A$and C are the functions of the parameter $t \epsilon IR$ this motion is called a one parameter motion and denoted by $B_1 = E/_{E'}$ [1].

Definition 2.4. The velocity vector of the point X with respect to the Euclidean plane E (moving space) i.e. the vectorial velocity of X while it is drawing its orbit in E is called relative velocity of the point X and denoted by V_r [1].

Definition 2.5. The velocity vector of the point $X$with respect to the fixed plane E' is called the absolute velocity of X and denoted by V_a. Thus we obtain the relation

(2.5) $$V_a = V_f + V_r$$

If X is a fixed point in the moving plane E, since $V_r = 0$, then we have $V_a = V_f$ The quality (2.5) is said to be the velocity law the motion $B_1 = E/_{E'}$ [3].

III. POLES OF ROTATING AND ORBIT

The point in which the sliding velocity V_f at each moment t of a fixed point X in E in the one-parameter homothetic motion $B_1 = E/_{E'}$ are fixed points in moving and fixed plane. These points are called the pole points of the motion.

Theorem 3.1. In a motion $B_1 = E/_{E'}$ whose angular velocity is non zero, there exists a unique point which is fixed in both planes at every moment t.

Proof. Since the point $X \in E$ is fixed in E then $V_r = 0$ and since X is also fixed in E' then $V_f = 0$. Hence for this type of points if $V_f = 0$ then

(3.1) $$\dot{B}X + \dot{C} = 0$$

and

(3.2) $$X = -\dot{B}^{-1}\dot{C}$$

Indeed,since

$$B = \begin{bmatrix} h\cos\varphi & -h\sin\varphi & 0 & 0 \\ h\sin\varphi & h\cos\varphi & 0 & 0 \\ 0 & 0 & h\cos\varphi & -h\sin\varphi \\ 0 & 0 & h\sin\varphi & h\cos\varphi \end{bmatrix}$$

and

$$\dot{B} = \begin{bmatrix} \dot{h}\cos\varphi - h\dot{\varphi}\sin\varphi & -\dot{h}\sin\varphi - h\dot{\varphi}\cos\varphi & 0 & 0 \\ \dot{h}\sin\varphi + h\dot{\varphi}\cos\varphi & \dot{h}\cos\varphi - h\dot{\varphi}\sin\varphi & 0 & 0 \\ 0 & 0 & \dot{h}\cos\varphi - h\dot{\varphi}\sin\varphi & -\dot{h}\sin\varphi - h\dot{\varphi}\cos\varphi \\ 0 & 0 & \dot{h}\sin\varphi + h\dot{\varphi}\cos\varphi & \dot{h}\cos\varphi - h\dot{\varphi}\sin\varphi \end{bmatrix}$$

then

(3.3) $C = [c_1 c_2 c_3 c_4]^T$

implies that

(3.4) $$\dot{C} = [\dot{c_1}\dot{c_2}\dot{c_3}\dot{c_4}]^T$$

and

(3.5) $$det\dot{B} = (\dot{h}^2 + h^2\dot{\varphi}^2)^2 \neq 0.$$

Thus $\dot{B}$ is regular and

$$\dot{B}^{-1} = \frac{1}{(\dot{h}^2 + h^2\dot{\varphi}^2)^2}\begin{bmatrix} K\cos\varphi - M\sin\varphi & -K\sin\varphi - M\cos\varphi & 0 & 0 \\ K\sin\varphi + M\cos\varphi & K\cos\varphi - M\sin\varphi & 0 & 0 \\ 0 & 0 & K\cos\varphi - M\sin\varphi & -K\sin\varphi - M\cos\varphi \\ 0 & 0 & K\sin\varphi + M\cos\varphi & K\cos\varphi - M\sin\varphi \end{bmatrix}$$

Hence there exists a unique solution $X$of the equation $V_f = 0$. This point X is called pole point in moving plane. For this reason (3.2) leads to

(3.6) $$X = -\dot{B}^{-1}\dot{C}$$

$$X = P = \frac{-1}{(\dot{h}^2 + h^2\dot{\varphi}^2)^2}\begin{bmatrix} \dot{c_1}(K\cos\varphi - M\sin\varphi) + \dot{c_2}(K\sin\varphi + M\cos\varphi) \\ -\dot{c_1}(K\sin\varphi + M\cos\varphi) + \dot{c_2}(K\cos\varphi - M\sin\varphi) \\ \dot{c_3}(K\cos\varphi - M\sin\varphi) + \dot{c_4}(K\sin\varphi + M\cos\varphi) \\ -\dot{c_3}(K\sin\varphi + M\cos\varphi) + \dot{c_4}(K\cos\varphi - M\sin\varphi) \end{bmatrix}$$

where $K = \dot{h}^3 + h^2\dot{h}\dot{\varphi}^2, M = h\dot{h}^2\dot{\varphi} + h^3\dot{\varphi}^3$and the pole point in the fixed plane is

(3.7) $$P^{'} = BP + C$$

setting these values in their planes and calculating we have

$$Y = P^{'} = \frac{1}{(\dot{h}^2 + h^2\dot{\varphi}^2)^2}\begin{bmatrix} -\dot{c_1}hK - \dot{c_2}hM \\ \dot{c_1}hM - \dot{c_2}hK \\ -\dot{c_3}hK - \dot{c_4}hM \\ \dot{c_3}hM - \dot{c_4}hK \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

or as a vector

(3.8)
$$Y = P^{'} = (W(-\dot{c_1}hK - \dot{c_2}hM) + c_1, W(\dot{c_1}hM - \dot{c_2}hK) + c_2, W(-\dot{c_3}hK - \dot{c_4}hM) + c_3, W(\dot{c_3}hM - \dot{c_4}hK) + c_4)$$

where $W = \frac{1}{(\dot{h}^2 + h^2\dot{\varphi}^2)^2}$.

Corollary 3.2. If $\varphi(t) = t$ then we obtain

$$P = X = \frac{-1}{(\dot{h}^2 + h^2)^4}\begin{bmatrix} \dot{c_1}(U\cos\varphi - V\sin\varphi) + \dot{c_2}(U\sin\varphi + V\cos\varphi) \\ -\dot{c_1}(U\sin\varphi + V\cos\varphi) + \dot{c_2}(U\cos\varphi - V\sin\varphi) \\ \dot{c_3}(U\cos\varphi - V\sin\varphi) + \dot{c_4}(U\sin\varphi + V\cos\varphi) \\ -\dot{c_3}(U\sin\varphi + V\cos\varphi) + \dot{c_4}(U\cos\varphi - V\sin\varphi) \end{bmatrix}$$

where $U = \dot{h}^3 + h^2\dot{h}, V = h\dot{h}^2 + h^3$.

Corollary 3.3. If $\varphi(t) = t$ and $h(t) = 1$, then we obtain

$$X = P = \begin{bmatrix} \dot{c_1} sin\varphi - \dot{c_2} cos\varphi \\ \dot{c_1} cos\varphi + \dot{c_2} sin\varphi \\ \dot{c_3} sin\varphi - \dot{c_4} cos\varphi \\ \dot{c_3} cos\varphi + \dot{c_4} sin\varphi \end{bmatrix}$$

Corollary 3.4. If $\varphi(t) = t$, then we obtain

$$Y = P' = \frac{-1}{(\dot{h}^2 + h^2)^4} \begin{bmatrix} -\dot{c_1} hU + \dot{c_2} hV \\ -\dot{c_1} hV + \dot{c_2} hU \\ \dot{c_3} hU + \dot{c_4} hV \\ -\dot{c_3} hV + \dot{c_4} hU \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

Corollary 3.5. If $\varphi(t) = t$ and $h(t) = 1$, then we obtain

(3.9) $$Y = P' = (-\dot{c_2} + c_1, \dot{c_1} + c_2, -\dot{c_4} + c_3, \dot{c_3} + c_4)$$

Here we assume that $\dot{\varphi}(t) \neq 0$ for all t. That is, angular velocity is not zero. In this case there exists a unique pole points in each of the moving and fixed planes of each moment t.

Definition 3.6. The point P = (p_1,p_2,p_3,p_4) is called the instantanious rotation center or the pole at moment t of the one parameter Euclidean motion $B_1 = {E}/{E'}$ [4].

Theorem 3.7. The following relation exists between the pole ray from the pole P to the point X, and the sliding velocity vector V_f at each moment t.

(3.10) $$\|V_f\| cos\varphi = \frac{\dot{h}}{h} \|P'Y\|$$

Proof. The pole point in the moving plane

(3.11) $$Y = BX + C,$$

implies that

(3.12) $$X = B^{-1}(Y - C),$$

(3.13) $$V_f = \dot{B}X + \dot{C},$$

and

(3.14) $$\dot{B}X + \dot{C} = 0$$

leads to

(3.15) $$X = P = -\dot{B}^{-1}\dot{C}$$

Now Let's find pole points in the fixed plane. Then we have from equation $Y = BX + C$

(3.16) $$Y = BX + C,$$

(3.17) $$Y = P' = B(-\dot{B}^{-1}\dot{C}) + C.$$

Hence, we get

(3.18) $$P' - C = -B\dot{B}^{-1}\dot{C},$$

(3.19) $$\dot{C} = -\dot{B}B^{-1}(P' - C)$$

If we substitute this values in the equation $V_f = \dot{B}X + \dot{C}$, we have $V_f = \dot{B}B^{-1}P'Y$. Now let us calculate the value of $\dot{B}B^{-1}P'Y$ here since $P'Y = (y_1 - p_1, y_2 - p_2, y_3 - p_3, y_4 - p_4)$, then

$$V_f = \left(\frac{\dot{h}}{h}(y_1 - p_1) - \dot{\varphi}(y_2 - p_2),\ \dot{\varphi}(y_1 - p_1) + \frac{\dot{h}}{h}(y_2 - p_2),\ \ \frac{\dot{h}}{h}(y_3 - p_3) - \dot{\varphi}(y_4 - p_4),\right.$$
$$\left.\dot{\varphi}(y_3 - p_3) + \frac{\dot{h}}{h}(y_4 - p_4)\right)$$

hence we obtain

(3.21) $$\langle V_f, P'Y\rangle = \frac{\dot{h}}{h}[(y_1 - p_1)^2 + (y_2 - p_2)^2 + (y_3 - p_3)^2 + (y_4 - p_4)^2]$$

(3.22) $$\langle V_f, P'Y\rangle = \frac{\dot{h}}{h}\|P'Y\|^2$$

on the other hand we know that

(3.23) $$\langle V_f, P'Y\rangle = \|V_f\|.\|P'Y\|.cos\varphi.$$

This from the equelities in (3.22) and (3.23) we have that

(3.24) $$\|V_f\| cos\varphi = \frac{\dot{h}}{h}\|P'Y\|$$

Corollary 3.8. The pole ray from the pole P to the point X, when the scalar matrix h is constant, is perpendicular to the sliding velocity vector V_f at each instant moment t.

Theorem 3.9. The length of the sliding velocity vector V_f is

(3.25)
$$\|V_f\| = \sqrt{\left(\left(\frac{\dot{h}}{h}\right)^2 + \dot{\varphi}^2\right)}\|P'Y\|$$

Proof.

$$V_f = \left(\frac{\dot{h}}{h}(y_1 - p_1) - \dot{\varphi}(y_2 - p_2),\ \dot{\varphi}(y_1 - p_1) + \frac{\dot{h}}{h}(y_2 - p_2),\ \ \frac{\dot{h}}{h}(y_3 - p_3) - \dot{\varphi}(y_4 - p_4),\right.$$
$$\left.\dot{\varphi}(y_3 - p_3) + \frac{\dot{h}}{h}(y_4 - p_4)\right)$$

hence

(3.27) $$\|V_f\| = \sqrt{\left(\left(\frac{\dot{h}}{h}\right)^2 + \dot{\varphi}^2\right)}\|P'Y\|$$

Corollary 3.10. If the scalar matrix is h is constant, then length of the sliding velocity vector is

(3.28) $$\|V_f\| = |\dot{\varphi}|\|P'Y\|$$

Corollary 3.11. There is a relation among the pole ray from the pole P to the point X, the sliding velocity vector V_f and angular velocity $\dot{\varphi}(t) \neq 0$ at each moment t.

(3.29) $$h(t) = exp(\int (cot\theta(t)\dot{\varphi}(t)dt)$$

proof. By the using of equations (3.24) are (3.27), we have

(3.30) $$\frac{1}{cos\theta}\left(\frac{\dot{h}}{h}\right) = \sqrt{\left(\left(\frac{\dot{h}}{h}\right)^2 + \dot{\varphi}^2\right)}$$

therefore we get

(3.31) $$h(t) = exp(\int (cot\theta(t)\dot{\varphi}(t)dt)$$

Definition 3.12. In Euclidean motion$B_1 = {}^{E}/_{E'}$ the geometric place of the pole points P in the moving plane E is called the moving pole curve of the motion $B_1 = {}^{E}/_{E'}$ and is denoted by (P). the geometric place of the pole points P in the fixed plane $E^{'}$is called fixed and is denoted by $P^{'}[2]$.

Theorem 3.13. The velocity on the curve (P) and $(P^{'})$ of every moment$t$of the rotating pol P which draws the pole curves in the fixed and moving planes are equal to each other. In other words, two curves are always tangent to each other.

Proof. The velocity of the point $X \in E$ while drawing the curve (P) is V_r and the velocity of this point while drawing the curve $(P^{'})$ is V_a. Since $V_f = 0$ then $V_a = V_r$

Definition 3.14. If two curves α and $\alpha^{'}$ are tangent to each other of each moment t and if length of the ways ds and $ds^{'}$of the point drawing these two curves at moment dt on these curves are the same then α and α′ are said to be revolving by sliding on each other. Here h is the coeffcient of rolling [2].

Theorem 3.15. In the one parameter planer Euclidean motion $B_1 = {}^{E}/_{E'}$ the moving pole curve (P) of the space E revolves by sliding on the fixed pole curve $(P^{'})$ of the space $E^{'}$.

Proof. Acording to the definition of ray element of a curve ray of (P) is $ds = \|V_r\|$ and those of (P) is $ds^{'} = \|V_a\|$. Since for (P) and $(P^{'})$, $V_a = V_r$ then $ds = hds^{'}$. According to this theorem we way define a Euclidean motion without mentioning the time. A Euclidean motion $B_1 = {}^{E}/_{E'}$ is obtained by a moving pol curve (P) of E revolving without sliding on a fixed pol curve $(P^{'})$.

Definition 3.16. Absolute accelaration vector of the point X with respect to the xed Euclidean plane $E^{'}$ is V_a. This vector is denoted by b_a . Since then $V_a = \dot{Y}$, then $b_a = \dot{V} = \ddot{Y}$ [1].

Definition 3.17. Let Xbe a fixed point the moving Euclidean plane E.The accelaration vector of the point X with respect to the fixed Euclidean plane $E^{'}$is called as sliding accelaration vector and denoted byb_f. Since in the accelaration of the sliding accelaration X is a fixed point of E, then $b_f = \dot{V_f} = \ddot{B}X + \ddot{C}$ [4, 5].

IV. ACCELERATIONS AND UNION OF ACCELERATIONS

Assume that the Euclidean motion $B_1 = {}^{E}/_{E'}$ of the moving euclidean plane E with respect to the fixed Euclidean plane $E^{'}$exists. In this motion, let us consider a point X moving with respect to the plane E, and thus moving respect to the plane $E^{'}$. We had obtained the velocity formulas concering the motion of X, now we will obtain the acceleration formules the acceleration of the point X.

Definition 4.1. The vector $b_r = \dot{V_r} = \ddot{B}X$ which is obtained by differenttiating the relative velocity vector $V_r = B\dot{X}$ of the point X with respect to the moving plane E is called the relative accelaration vector of X in E and denote by b_r. Since when taking the derivative X is considered as a moving point in E, the matrix A is taken as constant [6].

Theorem 4.2. Let X be a point in the moving Euclidean plane which moves with respect to a parameter t. Hence we have that

$$b_a = b_r + b_c + b_f,$$

Here , $b_c = 2\dot{B}\dot{X}$ is called Corilois acceleration.

Corollary 4.3. If a point $X \in E$ is constant,then the sliding acceleration of the point X is equal to the absolute acceleration of X.

Proof. Note that

$$V_a = \dot{B}X + B\dot{X} + \dot{C}$$

differentiating the both sides we have

$$\dot{V_a} = \ddot{B}X + 2\dot{B}\dot{X} + B\ddot{X} + \ddot{C} \tag{4.3}$$

since the point X is constant its derivativeis zero. Hence

(4.4)

$$\dot{V_a} = \ddot{B}X + \ddot{C} = b_f$$

Theorem 4.4. We have the following relation between the Coriolis acceleration vector b_c and relative velocity vector V_r

(4.5) $$\langle b_c, V_r \rangle = 2h\dot{h}(\dot{x_1}^2 + \dot{x_2}^2 + \dot{x_3}^2 + \dot{x_4}^2)$$

Proof. Since

(4.6) $$b_c = 2\dot{B}\dot{X} = (m\dot{x_1} - n\dot{x_2}, n\dot{x_1} + m\dot{x_2}, m\dot{x_3} - n\dot{x_4}, n\dot{x_3} + m\dot{x_4})$$

(4.7) $$V_r = B\dot{X} = (\dot{x_1}hcos\varphi - \dot{x_2}hsin\varphi, \dot{x_1}hsin\varphi + \dot{x_2}hcos\varphi, \dot{x_3}hcos\varphi - \dot{x_4}hsin\varphi, \dot{x_3}hsin\varphi + \dot{x_4}hcos\varphi)$$

(4.8) $$\langle b_c, V_r \rangle = 2h\dot{h}(\dot{x_1}^2 + \dot{x_2}^2 + \dot{x_3}^2 + \dot{x_4}^2)$$

where $m = \dot{h}cos\varphi - h\dot{\varphi}sin\varphi, n = \dot{h}sin\varphi + h\dot{\varphi}cos\varphi.$

Corollary 4.5. If h is a constant,then Coriolis acceleration b_c is perpendicular to the relative velocity vector V_r at each instant moment t.

V. FIRST AND SECOND ACCELERATION POLES

The solution of the equation $\dot{V_f} = 0$ gives the first order acceleration pole. $\dot{V_f} = \ddot{B}X + \ddot{C} = 0$ implies $X = P_1 = -\ddot{B}^{-1}\ddot{C}$.

Now calculating the matrices $-\ddot{B}^{-1}$ and $\ddot{C}$ and setting these in $X = P_1 = -\ddot{B}^{-1}\ddot{C}$ we obtain

$$X = P_1 = \frac{-1}{S}\begin{bmatrix} \ddot{c_1}(k_1cos\varphi - k_2sin\varphi) + \ddot{c_2}(k_2cos\varphi + k_1sin\varphi) \\ -\ddot{c_1}(k_2cos\varphi + k_1sin\varphi) + \ddot{c_2}(k_1cos\varphi - k_2sin\varphi) \\ \ddot{c_3}(k_1cos\varphi - k_2sin\varphi) + \ddot{c_4}(k_2cos\varphi + k_1sin\varphi) \\ -\ddot{c_3}(k_2cos\varphi + k_1sin\varphi) + \ddot{c_4}(k_1cos\varphi - k_2sin\varphi) \end{bmatrix}$$

Here, $P_1$is called first order pole curve in the moving plane. Denoting the pole curve in fixed plane by ${P'}_1$we get (5.1

$${P'}_1 = BP_1 + C$$

Hence

$$Y = {P'}_1 = \frac{-1}{(\dot{h}^2 + h^2\dot{\varphi}^2)^2}\begin{bmatrix} -\ddot{c_1}hk_1 - \ddot{c_2}hk_2 \\ \ddot{c_1}hk_2 - \ddot{c_2}hk_1 \\ -\ddot{c_3}hk_1 - \ddot{c_4}hk_2 \\ \ddot{c_3}hk_2 - \ddot{c_4}hk_1 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

where

$$\ddot{B} = \begin{pmatrix} D & -E & 0 & 0 \\ E & D & 0 & 0 \\ 0 & 0 & D & -E \\ 0 & 0 & E & D \end{pmatrix}$$

then

(5.2) $$C = [c_1 c_2 c_3 c_4]^T,$$

implies that

(5.3) $$\dot{C} = [\dot{c_1}\dot{c_2}\dot{c_3}\dot{c_4}]^T$$

and

(5.4) $$det\ddot{B} = (k_1^2 + k_2^2)^2 = S^2 \neq 0.$$

Thus $\ddot{B}$ is regular and

$$\ddot{B}^{-1}=\frac{1}{S}\begin{pmatrix} k_1cos\varphi-k_2sin\varphi & k_2cos\varphi+k_1sin\varphi & 0 & 0\\ -k_2cos\varphi-k_1sin\varphi & k_1cos\varphi-k_2sin\varphi & 0 & 0\\ 0 & 0 & k_1cos\varphi-k_2sin\varphi & k_2cos\varphi+k_1sin\varphi\\ 0 & 0 & -k_2cos\varphi-k_1sin\varphi & k_1cos\varphi-k_2sin\varphi \end{pmatrix}$$

where $D=(\ddot{h}-h\dot{\varphi}^2)cos\varphi+(-h\ddot{\varphi}-2\dot{h}\dot{\varphi})sin\varphi$, $E=\left(2\dot{h}\dot{\varphi}+h\ddot{\varphi}\right)cos\varphi+\left(\ddot{h}-h\dot{\varphi}^2\right)sin\varphi,$

$$k_1=\ddot{h}-h\dot{\varphi}^2, k_1=\ 2\dot{h}\dot{\varphi}+h\ddot{\varphi}$$

Corollary 5.1. If $\varphi(t)=t$, then we obtain

$$X=P_1=\frac{-1}{(\dot{h}-h)^2+(2h)^2}\begin{bmatrix} \ddot{c_1}(Fcos\varphi-Gsin\varphi)+\ddot{c_2}(Gcos\varphi+Fsin\varphi)\\ -\ddot{c}_1(Gcos\varphi+Fsin\varphi)+\ddot{c_2}(Fcos\varphi-Gsin\varphi)\\ \ddot{c_3}(Fcos\varphi-Gsin\varphi)+\ddot{c_4}(Gcos\varphi+Fsin\varphi)\\ -\ddot{c}_3(Gcos\varphi+Fsin\varphi)+\ddot{c_4}(Fcos\varphi-Gsin\varphi \end{bmatrix}$$

where $F=\ddot{h}-h, G=2\dot{h}$.

Corollary 5.2. If $\varphi(t)=t$ ve $h(t)=1$, then we obtain

$$X=P_1=\begin{bmatrix} -\ddot{c_1}\ cos\varphi-\ddot{c_2}sin\varphi\\ \ddot{c_1}sin\varphi-\ddot{c_2}cos\varphi)\\ -\ddot{c}_3\ cos\varphi-\ddot{c_4}sin\varphi\\ \ddot{c_3}\ sin\varphi-\ddot{c_4}cos\varphi \end{bmatrix}$$

Corollary 5.3. If $\varphi(t)=t$, then we obtain

$$Y=P_1^{'}=\frac{1}{(\dot{h}-h)^2+(2h)^2}\begin{bmatrix} -\ddot{c_1}hF-\ddot{c_2}hG\\ \ddot{c_1}hG-\ddot{c_2}hF\\ \ddot{c_3}\ hF-\ddot{c_4}hG\\ \ddot{c_3}\ hG-\ddot{c_4}hF \end{bmatrix}+\begin{bmatrix} c_1\\ c_2\\ c_3\\ c_4 \end{bmatrix}$$

Corollary 5.4. If $\varphi(t)=t$ ve $h(t)=1$, then we obtain

(5.5) $$Y=P_1^{'}=(\ddot{c}_1+c_1,-\ddot{c}_2+c_2,-\ddot{c}_3+c_3,-\ddot{c}_4+c_4)$$

The solution of the equation $\ddot{V_f}=0$ gives the seond order acceleration pole.
$\ddot{V_f}=\ddot{B}X+\ddot{C}=0$ implies $X=-\ddot{B}^{-1}\ddot{C}$. Now calculating the matrices $-\ddot{B}^{-1}$ and $\ddot{C}$ and setting these in $X=P_2=-\ddot{B}^{-1}\ddot{C}$ we get

$$X=P_2=\frac{-1}{R}\begin{bmatrix} \dddot{c}_1(acos\varphi-bsin\varphi)+\dddot{c}_2(bcos\varphi+asin\varphi)\\ -\dddot{c}_1(bcos\varphi+asin\varphi)+\dddot{c}_2(acos\varphi-bsin\varphi)\\ \dddot{c}_3(acos\varphi-bsin\varphi)+\dddot{c}_4(bcos\varphi+asin\varphi)\\ -\dddot{c}_3(bcos\varphi+asin\varphi)+\dddot{c}_4(acos\varphi-bsin\varphi) \end{bmatrix}$$

The pole curve in the fixed plane is obtained as

$$P_2^{'}=\left(\frac{-1}{R}(\dddot{c}_1ah+\dddot{c}_2bh)+c_1,\frac{-1}{R}(\dddot{c}_2ah-\dddot{c}_1bh)+c_2,\frac{-1}{R}(\dddot{c}_3ah+\dddot{c}_4bh)+c_3,\right.$$

$$\left.\frac{-1}{R}(\dddot{c}_3ah-\dddot{c}_4bh)+c_4\right)$$

where

$$\dddot{B}=\begin{pmatrix} \dot{D} & -\dot{E} & 0 & 0\\ \dot{E} & \dot{D} & 0 & 0\\ 0 & 0 & \dot{D} & -\dot{E}\\ 0 & 0 & \dot{E} & \dot{D} \end{pmatrix}$$

where $k_1=\ddot{h}-h\dot{\varphi}^2, k_2=\ 2\dot{h}\dot{\varphi}+h\ddot{\varphi}$

$$D=\left(\ddot{h}-h\dot{\varphi}^2\right)cos\varphi+\left(-h\ddot{\varphi}-2\dot{h}\dot{\varphi}\right)sin\varphi=k_1cos\varphi+k_2sin\varphi$$

$$E=\left(2\dot{h}\dot{\varphi}+h\ddot{\varphi}\right)cos\varphi+\left(\ddot{h}-h\dot{\varphi}^2\right)sin\varphi=k_2cos\varphi+k_1sin\varphi$$

$$\dot{D} = (\dot{k}_1 - k_2\dot{\varphi})cos\varphi + (-k_1\dot{\varphi} - \dot{k}_2)sin\varphi = acos\varphi - bsin\varphi$$
$$\dot{E} = (\dot{k}_2 + k_1\dot{\varphi})cos\varphi + (\dot{k}_1 - k_2\dot{\varphi})sin\varphi = bcos\varphi + asin\varphi$$
$$a = \dot{k}_1 - k_2\ddot{\varphi}, \quad b = \dot{k}_2 + k_1\dot{\varphi}$$

(5.7) $det\ddot{B} = (a^2 + b^2)^2 = R^2 \neq 0$. Thus $\ddot{B}$ is regular and

$$\ddot{B}^{-1} = \frac{1}{R}\begin{pmatrix} acos\varphi - bsin\varphi & bcos\varphi + asin\varphi & 0 & 0 \\ -(bcos\varphi + asin\varphi) & acos\varphi - bsin\varphi & 0 & 0 \\ 0 & 0 & acos\varphi - bsin\varphi & bcos\varphi + asin\varphi \\ 0 & 0 & -(bcos\varphi + asin\varphi) & acos\varphi - bsin\varphi \end{pmatrix}$$

Corollary 5.5. If $\varphi(t) = t$ then, we obtain

$$X = P_2 = \frac{-1}{T_3}\begin{bmatrix} \ddot{c}_1(T_1cos\varphi - T_2sin\varphi) + \ddot{c}_2(T_2cos\varphi + T_1sin\varphi) \\ -\ddot{c}_1(T_2cos\varphi + T_1sin\varphi) + \ddot{c}_2(T_1cos\varphi - T_2sin\varphi) \\ \ddot{c}_3(T_1cos\varphi - T_2sin\varphi) + \ddot{c}_4(T_2cos\varphi + T_1sin\varphi) \\ -\ddot{c}_3(T_2cos\varphi + T_1sin\varphi) + \ddot{c}_4(T_1cos\varphi - T_2sin\varphi) \end{bmatrix}$$

where $T_1 = \ddot{h} - 3\dot{h}$, $T_2 = 3\ddot{h} - h$, $T_3 = (\ddot{h} - 3\dot{h})^2 + (3\ddot{h} - h)^2$ dir.

Corollary 5.6. If $\varphi(t) = t$ ve $h(t) = 1$, then we obtain

$$P_2 = X = \begin{bmatrix} -\ddot{c}_1cos\varphi - \ddot{c}_2sin\varphi \\ -\ddot{c}_1sin\varphi - \ddot{c}_2cos\varphi \\ -\ddot{c}_3cos\varphi - \ddot{c}_4sin\varphi \\ \ddot{c}_3sin\varphi - \ddot{c}_4cos\varphi \end{bmatrix}$$

Corollary 5.7. If $\varphi(t) = t$, then we obtain

$$Y = P'_2 = \frac{-1}{T_3}\begin{bmatrix} \ddot{c}_1hT_1 + \ddot{c}_2hT_2 \\ \ddot{c}_2hT_1 - \ddot{c}_1hT_2 \\ \ddot{c}_3hT_1 - \ddot{c}_4hT_2 \\ \ddot{c}_3hT_1 - \ddot{c}_4hT_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

Corollary 5.8. If$\varphi(t) = t$ ve $h(t) = 1$, then we obtain

(5.8) $$Y = P'_2 = (-\ddot{c}_2 + c_1, \ddot{c}_1 + c_2, -\ddot{c}_4 + c_3, \ddot{c}_4 + c_4).$$

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