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Mathematical Modelling Of Cattle Culling Using Least Square Method

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ABSTRACT. *In this paper, a population of Cattle was used to construct a Malthusian model using the least square method. The differential equation in Malthusian model was modified to include culling of E(t) cattle per year. The paper also discussed the strategy of culling a fixed number of cattle q for every year and concluded by showing that there is a critical value $\bar{q}$ such thatif $q > \bar{q}$ the population will be extinct in a finite period of time, and if $q < \bar{q}$, the population will increase, butchoosing a value of $q = \bar{q}$, the population will stay constant.*

KEYWORDS: *Cattle, Culling, Malthusian model, Laplace transform, Population, Ranch.*

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I. INTRODUCTION

Cattle is a livestock commonly grazed in Northern Nigeria which is a large ruminant animal with horns and cloven hoofs, domesticated for meat or milk or beasts of burden, cows and oxen [1]. Considering the theoretical situation where the owner of a ranch in Nigeria has to cull a number of the population of cattle to prevent overgrazing. The owner is however, not sure as to how many cattle must be removed and when they must be removed, furthermore, the owner is worried about the long - term effect of such a culling on their population. Due to the unsureness of the owner, we were approached as experts on biological models, to model the situation mathematically. Table 1 provides the observed population of the cattle at the beginning of each year.

Table 1. Yearlycattle population

Year	Population
2011	260
2012	370
2013	500
2014	680
2015	950

II. FORMULATION OF MALTHUSIAN MODEL OF THE CATTLE POPULATION

Suppose we make the assumption that the rate of growth of the population of cattle is proportional to the population of cattle then we have

$$\frac{dR}{dt} = \alpha R(t) \quad (1)$$

where $R(t)$ is the estimated population of cattle at time t and α is the intrinsic growth rate i.e the constant of proportionality of the population [2]. Solving the separable first order ordinary differential (1), we arrive at the well-known result

$$R(t) = R_0 e^{\alpha t} \quad (2)$$

Equation (2) is known as the Malthusian population model where R_0 is the estimated initial population at $t = 0$; i.e $R(0) = R_0$. Manipulating (2) we arrive at the logarithmic equation,

$$ln R = ln R_0 + \alpha t \quad (3)$$

Thus applying the method of least squares by substituting our data from Table 1 into (3) we have

$$ln260 = lnR_0 + \alpha(0)$$

$ln370 = lnR_0 + \alpha(1)$
$ln500 = lnR_0 + \alpha(2)$
$ln680 = lnR_0 + \alpha(3)$
$ln950 = lnR_0 + \alpha(4)$

where the 2011 is taken to be year *0*, and *t* increases by unit per year. This system can be written as the following matrix equation.

$$\begin{bmatrix} ln260 \\ ln370 \\ ln500 \\ ln680 \\ ln950 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} lnR_0 \\ \alpha \end{bmatrix}$$

$$\Rightarrow b = Ax \qquad (4)$$

Thus multiplying by A^T on both sides yields,

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}^T \begin{bmatrix} ln260 \\ ln370 \\ ln500 \\ ln680 \\ ln950 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} lnR_0 \\ \alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 10 \\ 10 & 30 \end{bmatrix} \begin{bmatrix} lnR_0 \\ \alpha \end{bmatrix} = \begin{bmatrix} ln(3.10726\text{x}10)^{13} \\ ln(2.3689881\text{x}10)^{28} \end{bmatrix}$$

Applying row reduction reduces the matrix to

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} lnR_0 \\ \alpha \end{bmatrix} = \begin{bmatrix} 5.57344 \\ 0.320015 \end{bmatrix}$$

Hence we have α = 0.320015 and lnR_o = 263.3384271 and

$$R(t) = 263.3384271e^{0.320015t} \qquad (5)$$

For the rest of this model we will take R_0 and α accurate to four and two decimal places respectively.

III. INCLUSION OF CULLING TO THE MALTHUSIAN MODEL

Defining *E(t)* be the number of cattle culled per year starting in 2015, we can alter (1) subtract the culled cattle and form the following model.

$$\frac{dR}{dt} = \alpha R(t) - E(t) \qquad (6)$$

Applying the Laplace transform to (6), rearranging and then applying the Inverse Laplace transform we have the following steps,

$$\frac{dR}{dt}\}(s) = \ell\{\alpha R(t) - E(t)\}(s)$$

$$s\ell\{R\}(s) - R(0) = \alpha\ell\{R\}(s) - \ell\{E(t)\}(s)$$

$$\ell\{R\}(s) = \frac{R_0}{s-\alpha} - \frac{\ell\{E(t)\}(s)}{s-\alpha}$$

$$R(t) = \ell^{-1}\{\frac{R_0}{s-\alpha} - \ell^{-1}\{\frac{\ell\{E(t)\}(s)}{s-\alpha}\}$$

$$R(t) = R_0e^{\alpha t} - \ell^{-1}\{\frac{\ell\{E(t)\}(s)}{s-\alpha}\}$$

Substituting values for R_0 and α gives

$$R(t) = 263.3384e^{0.32t} = \ell^{-1}\{\frac{\ell\{E(t)\}(s)}{s-0.32}\} \qquad (7)$$

for $t > 4$ as the culling begins after 2015, the fifth year. Suppose that a strategy is employed ofCulling a fixed number of cattle q; i.e $E(t) = q$. But the Laplace transform of a constant h is $\frac{h}{s}$ (7) becomes

$$R(t)= 263.3384e^{0.32t} - \ell^{-1}\{ \frac{\ell\{q\}(s)}{s-0.32} \}$$

$$= 263.3384e^{0.32t} = \ell^{-1}\{ \frac{q}{s(s-0.32)} \}$$

$$= 263.3384e^{0.32t} = \frac{q}{0.32}\ \ell^{-1}\{ \frac{1}{s-0.32} - \frac{1}{s} \}$$

$$R(t)= 263.3384e^{0.32t} = \frac{q}{0.32} (e^{0.32t} - 1) \qquad (8)$$

Thus, combining the models for $0 \le t \le 4$ and $t > 4$ yields the following equation

$$R(t)= 263.3384e^{0.32t} = \frac{q}{0.32} (e^{0.32t} - 1) t>4 \qquad (9)$$

which can be written alternatively in terms of unit- step functions as

$$R(t)= 263.3384e^{0.32t} = u(t-4)\frac{q}{0.32} (e^{0.32t} - 1) \qquad (10)$$

III. EXTINCTION OR OVERGRAZING

Given that q is fixed, we want to know the number q of cattle to tell the owner of the ranch to cullper year to keep a constant population and avoid an increase in population causing over grazing oran extinct population over a fixed time [3]. Hence, we must calculate a critical value for q, namely, $\bar{q}$ such that if q > $\bar{q}$ or q < $\bar{q}$ the population of cattle will be extinct in a finite period of time orthe population of cattle will continue to increase. Applying Laplace transformations across (10),we have the following steps

$$\ell\{R\}(s)= \ell\{263.3384e^{0.32t} = u(t-4)\frac{q}{0.32} (e^{0.32(t-4)} - 1)\}$$

$$= \ell\{263.3384e^{0.32t}\} - \ell\{u(t-4)\frac{q}{0.32} (e^{0.32(t-4)} - 1)\}$$

$$= \frac{263.3384}{s-0.32} - \frac{e^{-4s}}{s(s-0.32)}$$

Multiplying by $s - 0.32$ and rearranging, yields

$$s\ell\{R\}(s) - 263.3384 = 0.32\ell\{R\}(s) - \frac{q}{s}e^{-4s}$$

$$\ell\{\frac{dR}{dt}\}(s) = 0.32\ell\{R\}(s) - \frac{q}{s}e^{-4s} \qquad (11)$$

Since R_0 = 263:3384. To find a critical point $R(t)$ and the point at which

$$\frac{dR}{dt} = 0 \qquad (12)$$

Applying Laplace to both sides of (12), we obtain

$$\ell\{\frac{dR}{dt}\}(s) = 0 \qquad (13)$$

Substituting this into (11); rearranging for $\frac{q}{s}$ and applying the Inverse Laplace transform [4], we arrive at the following steps

$0.32\ell\{R\}(s) - \frac{q}{s}e^{-4s} = 0$

$$\Rightarrow \frac{q}{s} = 0.32\ell\{R\}(s)\, e^{4s}$$

$$\ell^{-1}\{\frac{q}{s}\} = 0.32\ \ell^{-1}\{0.32\ell\{R\}(s)\, e^{4s}\}$$

$\Rightarrow q = 0.32R(t+4)u(t+4)$

By employing the second shift theorem [4] and substituting with *R(t+4)* we have:

$q = 0.32u(t+4)[\ 263.3384e^{0.32(t+4)} - u(t)(\ \frac{q}{0.32}\ ^{0.32t} - 1)]$

substituting $t = 0$ (since we want the critical value at *R(t)* which is defined to be $\bar{q}$) In fact we have the following result for $\bar{q}$

$\bar{q} = 0.32u(0+4)[\ 263.3384e^{0.32(0+4)} - u(0)(\ \frac{q}{0.32}\ ^{0.32(0)} - 1)]$

$= 0.32 \times 263.3384e^{0.32(4)}$

$= 303.0836722$

From figures 1, 2 and 3 whose values for q are 303,304 and 303, 0836722 respectively, we are able to distinguish the different behaviors of the estimated population for which time $t \to \infty$ In fact, if we choose a value q > $\bar{q}$, as we have in figure (1), we are able to observe that estimated population *R(t)* of cattle per year *t* becomes extinct in a fixed period of time. By inspection of the Figure 1, if we suggest to the owner of the ranch to cull 304 cattle per year, his cattle population will be extinct in approximately 22 years.

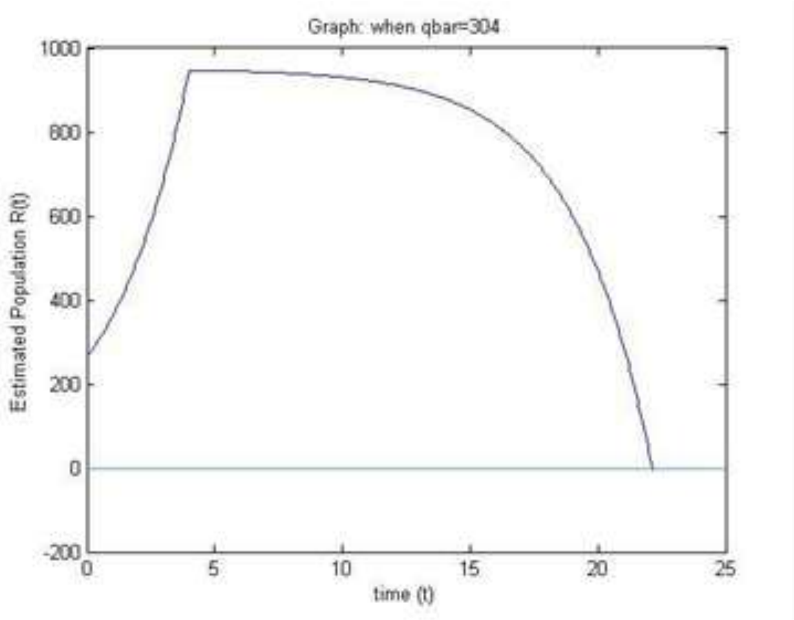

Figure 1:A plot of the approximate cattle population for q = 304 > $\bar{q}$

Then, choosing a value q < $\bar{q}$, as we have in figure2, we are able to observe that the estimated population *R(t)* of cattle per year *t* becomes exponentially large as $t \to \infty$. Thus if we were to advice the owner of the ranch to cull 303 cattle per year, he would experience a continuous increase in his estimated population for $t \to \infty$. This would cause overgrazing which we are trying to avoid and thus q = 303 would be a poor choice for a number of cattle to be culled per year.

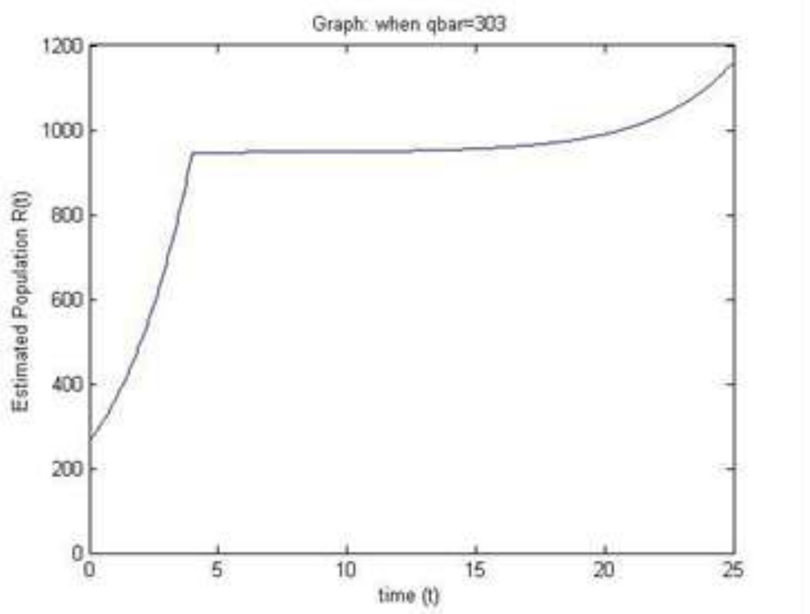

Figure 2: A plot of the approximate cattle population for q = 303< $\bar{q}$:

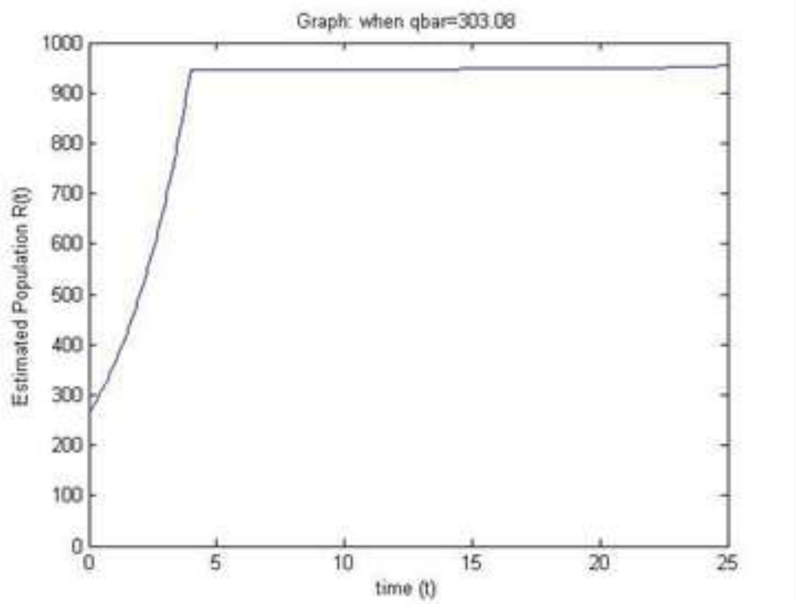

Figure 3: A plot of the approximate cattle population for q = 303.0836722< $\bar{q}$

IV. CONCLUSION

It is clear that, choosing a value of q = $\bar{q}$, as we have in figure 3, enables the population to be constant for $t \rightarrow \infty$. Although, this may be the optimal choice, but it is impossible to cull only part of a herd and hence the number of cattle to be culled must be an integer and not an optimal integer in order not to experience overgrazing or extinction of cattle population per year.

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