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Edge Vertex Prime Labeling forWheel, Fan and Friendship Graph

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ABSTRACT:*In this paper, we investigate edge vertex prime labeling of some graphs. We proved that wheel graph, fan graph and friendship graphs are edge vertex prime.*
KEYWORDS: *Edge-vertex prime labeling, fan graph, friendship graph,graph labeling, prime labeling, wheel graph.*

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I. INTRODUCTION

In this paper, all graphs $G = \big(V(G), E(G)\big)$are simple, undirected and finite. The symbol $V(G)$ and $E(G)$ will denote the vertex set and edge set of the graph G. $|V(G)|$and$|E(G)|$ symbols will denote number of vertices and number of edges respectively. For various graph theoretic notation and terminology, I follow J. Gross and J. Yellen [1] and for Number theory D Burton [2]. In this paper, I have proved that wheel graph, fan graph and friendship graph are edge vertex prime.

Definition 1.1:
Graph labeling is an assignment of integers either to the vertices or edges or both subject to certain conditions. A dynamic survey of graph labeling is regularly updated by J Gallian [3].

Definition 1.2:
A bijection function $f: V(G) \to \{1, 2, \dots, n\}$ is called *prime labeling* if for each edge $e = uv$, $gcd\big(f(u),$ *fv=1.*
A graph which admits prime labeling is called a prime graph.
The notion of a prime labeling was introduced by R Entringer and was discussed in a paper by Tout. A(1982 P 365 – 368)[4].

Definition 1.3:
A bijection function $f: V(G) \cup E(G) \to \{1, 2, \dots, |V(G) \cup E(G)|\}$ is called *edgevertexprime labeling* if for any edge $e = uv$; $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.
A graph which admits prime labeling is called an edge vertex prime graph.
Edge vertex prime labeling was introduced by R. Jagadesh and J. BaskarBabujee[5], they proved the existence of the edge vertex prime labelingfor paths, cycles and star-$K_{1,n}$.

Definition 1.4:
Wheel graph $W_n$is obtained by joining all vertices of a cycle C_n to a further vertex, called center.
$W_n = K_1 + C_n$, $|V(G)| = n + 1$ and $|E(G)| = 2n$
Definition 1.5:
Fan graph $f_n, n \geq 2$obtained by joining all vertices of a path P_n to a further vertex, called center.
$f_n = K_1 + P_n$, $|V(G)| = n + 1$ and $|E(G)| = 2n - 1$

Definition 1.6:
Friendship graph F_n, is a graph which consists of n −triangles with a common vertex.
$|V(G)| = 2n + 1$and$|E(G)| = 3n$

II. MAIN RESULTS

Theorem 2.1: Wheel graph is an edge vertex prime.
Proof: Let $G = W_n$ be the graph.

$V(G) = \{v, v_1, v_2, \dots\ v_n\}$and
$E(G) = \{vv_i / 1 \le i \le n\} \cup \{v_i v_{i+1} / 1 \le i \le n\}$
and a bijective function $f: V(G) \cup E(G) \to \{1, 2, \dots, 3n + 1\}$ is defined as follows:

Case-1: When n is even.

$f(v) = 1$

$$f(v_i) = \begin{cases} 3i & ;\ i \text{ is odd} \\ 3i - 1 & ;\ i \text{ is even} \end{cases}$$

$$f(vv_i) = \begin{cases} 3i - 1 & ;\ i \text{ is odd} \\ 3i & ;\ i \text{ is even} \end{cases}$$

$$f(v_i v_{i+1}) = 3i + 1;\ \forall i$$

Now our claims are (1) $f(v), f(v_i)$ and $f(vv_i)$ are pairwise relatively prime.
(2) $f(v_i), f(v_i v_{i+1})$ and $f(v_{i+1})$ are pairwise relatively prime.

(1) $$gcd\big(f(v), f(v_i)\big) = \begin{cases} gcd(1, 3i) & ;\ i \text{ is odd} \\ gcd(1, 3i - 1) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

$$gcd\big(f(v), f(vv_i)\big) = \begin{cases} gcd(1, 3i - 1) & ;\ i \text{ is odd} \\ gcd(1, 3i) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

and $$gcd\big(f(v_i), f(vv_i)\big) = \begin{cases} gcd(3i, 3i - 1) & ;\ i \text{ is odd} \\ gcd(3i - 1, 3i) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

(2) $$gcd\big(f(v_i), f(v_{i+1})\big) = \begin{cases} gcd(3i, 3(i + 1) - 1) & ;\ i \text{ is odd} \\ gcd\big(3i - 1, 3(i + 1)\big) & ;\ i \text{ is even} \end{cases}$$

$$= \begin{cases} gcd(3i, 3i + 2) & ;\ i \text{ is odd} \\ gcd(3i - 1, 3i + 3) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

$$gcd\big(f(v_i), f(v_i v_{i+1})\big) = \begin{cases} gcd(3i, 3i + 1) & ;\ i \text{ is odd} \\ gcd(3i - 1, 3i + 1) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

and $$gcd\big(f(v_{i+1}), f(v_i v_{i+1})\big) = \begin{cases} gcd(3(i + 1) - 1, 3i + 1) & ;\ i \text{ is odd} \\ gcd(3(i + 1), 3i + 1) & ;\ i \text{ is even} \end{cases}$$

$$= \begin{cases} gcd(3i + 2, 3i + 1) & ;\ i \text{ is odd} \\ gcd(3i + 3, 3i + 1) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.

Case-2: When n is odd.

$f(v) = 1$

$$f(v_i) = \begin{cases} 3i & ;\ i = 1, 3, 5, \dots, n - 2 \\ 3i - 1 & ;\ i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(vv_i) = \begin{cases} 3i - 1 & ;\ i \text{ is odd} \\ 3i & ;\ i \text{ is even} \end{cases}$$

$f(v_i v_{i+1}) = 3i + 1; \quad i = 1, 2, 3, \dots, n-2$

$$f(v_n) = 3n - 2$$

$$f(v_{n-1} v_n) = 3n$$

Now our claims are (1) $f(v), f(v_i)$ and $f(vv_i)$ are pairwise relatively prime.

(2) $f(v_i), f(v_i v_{i+1})$ and $f(v_{i+1})$ are pairwise relatively prime.

(1) $gcd\big(f(v), f(v_i)\big) = \begin{cases} gcd(1, 3i) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(1, 3i-1) & ; \quad i = 2, 4, 6, \dots, n-1 \\ gcd(1, 3n-2) & ; \quad i = n \end{cases}$

$$= 1$$

$$gcd\big(f(v), f(vv_i)\big) = \begin{cases} gcd(1, 3i-1) & ; \ i \ is \ odd \\ gcd(1, 3i) & ; i \ is \ even \end{cases}$$

$$= 1$$

and$gcd\big(f(v_i), f(vv_i)\big) = \begin{cases} gcd(3i, 3i-1) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(3i-1, 3i) & ; \quad i = 2, 4, 6, \dots, n-1 \\ gcd(3n-2, 3n+1) & ; \quad i = n \end{cases}$

$$= 1$$

(2) $gcd\big(f(v_i), f(v_{i+1})\big) = \begin{cases} gcd(3i, 3i+2) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(3i-1, 3i+3) & ; \quad i = 2, 4, 6, \dots, n-3 \\ gcd(3i-1, 3n-2) & ; \quad i = n-1 \end{cases}$

$$= \begin{cases} gcd(3i, 3i+2) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(3i-1, 3i+3) & ; \quad i = 2, 4, 6, \dots, n-3 \\ gcd(3i-1, 3i+1) & ; \quad i = n-1 \end{cases}$$

$$= 1$$

$$gcd\big(f(v_i), f(v_i v_{i+1})\big) = \begin{cases} gcd(3i, 3i+1) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(3i-1, 3i+1) & ; \quad i = 2, 4, 6, \dots, n-3 \\ gcd(3i-1, 3n) & ; \quad i = n-1 \end{cases}$$

$$= \begin{cases} gcd(3i, 3i+1) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(3i-1, 3i+1) & ; \quad i = 2, 4, 6, \dots, n-3 \\ gcd\big(3i-1, 3(i+1)\big) & ; \quad i = n-1 \end{cases}$$

$$= 1$$

and $gcd\big(f(v_{i+1}), f(v_i v_{i+1})\big) = \begin{cases} gcd(3(i+1)-1, 3i+1) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(3(i+1), 3i+1) & ; \quad i = 2, 4, 6, \dots, n-3 \\ gcd(3n-2, 3n) & ; \quad i = n-1 \end{cases}$

$$= \begin{cases} gcd(3i+2, 3i+1) & ; \quad i = 1, 3, 5, \dots, n-2 \\ gcd(3i+3, 3i+1) & ; \quad i = 2, 4, 6, \dots, n-3 \\ gcd(3i+1, 3i+3) & ; \quad i = n-1 \end{cases}$$

$$= 1$$

Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.
Hence W_n is an edge vertex prime graph.

Illustration 2.1: Edge vertex prime labeling for W_{28}.

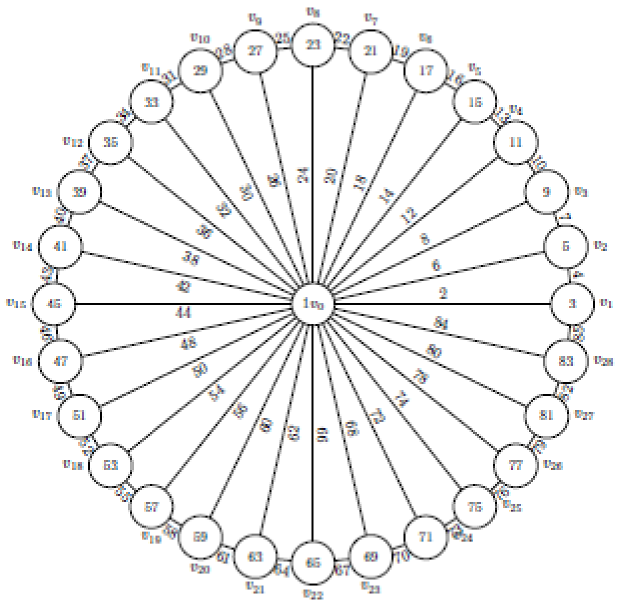

Fig. 2.1W_{28}

Illustration 2.2: Edge vertex prime labeling for W_{27}.

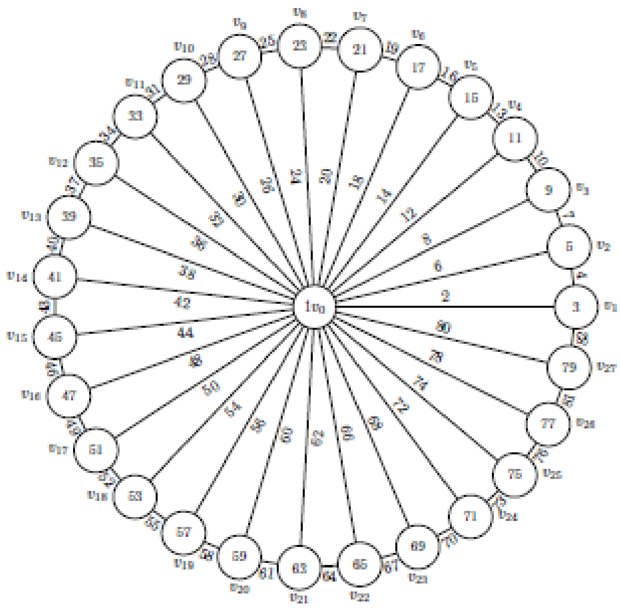

Fig.2.2 W_{27}.

Theorem 2.2: Fan graph is an edge vertex prime

Proof: Let $G = f_n, n \geq 2$ be the graph.

$V(G) = \{v, v_1, v_2, \dots\ v_n\}$and

$E(G) = \{vv_i/1 \leq i \leq n\} \cup \{v_i v_{i+1}\ /1 \leq i \leq n-1\}$

and a bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 3n\}$ is defined as follows:

$f(v) = 1$

$$f(v_i) = \begin{cases} 3i & ;\ i\ is\ odd \\ 3i-1 & ;\ i\ is\ even \end{cases}$$

$$f(vv_i) = \begin{cases} 3i-1 & ;\ i\ is\ odd \\ 3i & ;\ i\ is\ even \end{cases}$$

$f(v_i v_{i+1}) = 3i + 1;\ \forall i$

Now our claims are (1) $f(v), f(v_i)$ and $f(vv_i)$ are pairwise relatively prime.

(2) $f(v_i), f(v_i v_{i+1})$ and $f(v_{i+1})$ are pairwise relatively prime.

(1) $gcd\big(f(v), f(v_i)\big) = \begin{cases} gcd(1, 3i) & ;\ i\ is\ odd \\ gcd(1, 3i-1) & ; i\ is\ even \end{cases}$

$= 1$

$gcd\big(f(v), f(vv_i)\big) = \begin{cases} gcd(1, 3i-1) & ;\ i\ is\ odd \\ gcd(1, 3i) & ; i\ is\ even \end{cases}$

$= 1$

and $gcd\big(f(v_i), f(vv_i)\big) = \begin{cases} gcd(3i, 3i-1) & ;\ i\ is\ odd \\ gcd(3i-1, 3i) & ; i\ is\ even \end{cases}$

$= 1$

(2) $gcd\big(f(v_i), f(v_{i+1})\big) = \begin{cases} gcd(3i, 3(i+1)-1) & ;\ i\ is\ odd \\ gcd\big(3i-1, 3(i+1)\big) & ; i\ is\ even \end{cases}$

$= \begin{cases} gcd(3i, 3i+2) & ;\ i\ is\ odd \\ gcd(3i-1, 3i+3) & ; i\ is\ even \end{cases}$

$= 1$

$gcd\big(f(v_i), f(v_i v_{i+1})\big) = \begin{cases} gcd(3i, 3i+1) & ;\ i\ is\ odd \\ gcd(3i-1, 3i+1) & ; i\ is\ even \end{cases}$

$= 1$

and $gcd\big(f(v_{i+1}), f(v_i v_{i+1})\big) = \begin{cases} gcd(3(i+1)-1, 3i+1) & ;\ i\ is\ odd \\ gcd(3(i+1), 3i+1) & ; i\ is\ even \end{cases}$

$= \begin{cases} gcd(3i+2, 3i+1) & ;\ i\ is\ odd \\ gcd(3i+3, 3i+1) & ; i\ is\ even \end{cases}$

$= 1$

Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence fan graph is an edge vertex prime.

Illustration 2.3: Edge vertex prime labeling for f_{29}.

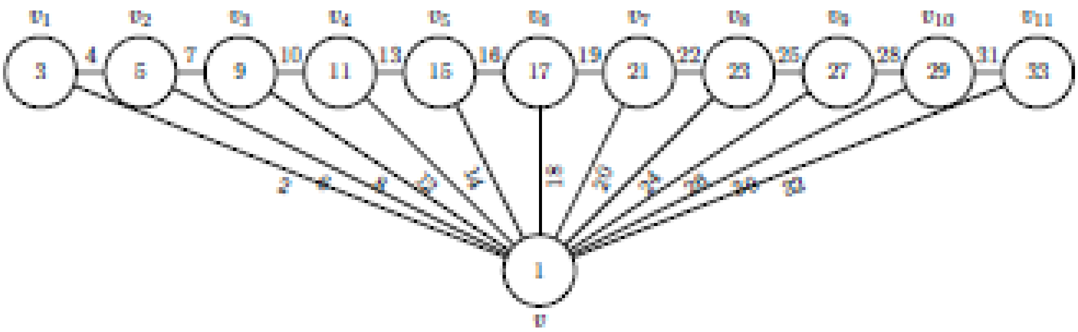

Fig. 2.3 f_{29}

Theorem 2.3: Friendship graph is an edge vertex prime

Proof: Let $G = F_n$ be the graph.

$V(G) = \{v, v_1, v_2, \ldots\ v_{2n}\}$and

$E(G) = \{vv_i / 1 \le i \le 2n\} \cup \{v_{2i-1}v_{2i}\ / 1 \le i \le n\}$

and a bijective function $f: V(G) \cup E(G) \to \{1, 2, \ldots, 5n + 1\}$ is defined as follows:

$f(v) = 1$

$$f(v_{2i-1}) = \begin{cases} 5i-2 & ;\ i \text{ is odd} \\ 5i-3 & ;\ i \text{ is even} \end{cases}$$

$$f(v_{2i}) = \begin{cases} 5i & ;\ i \text{ is odd} \\ 5i+1 & ;\ i \text{ is even} \end{cases}$$

$$f(vv_{2i-1}) = \begin{cases} 5i-3 & ;\ i \text{ is odd} \\ 5i-2 & ;\ i \text{ is even} \end{cases}$$

$$f(vv_{2i}) = \begin{cases} 5i+1 & ;\ i \text{ is odd} \\ 5i & ;\ i \text{ is even} \end{cases}$$

$$f(v_{2i-1}v_{2i}) = 5i-1 \quad ;\ \forall i$$

Now our claims are (1) $f(v), f(v_{2i-1})$ and $f(vv_{2i-1})$ are pairwise relatively prime.
(2) $f(v), f(v_{2i})$ and $f(vv_{2i})$ are pairwise relatively prime.
(3) $f(v_{2i-1}), f(v_{2i-1}v_{2i})$ and $f(v_{2i})$ are pairwise relatively prime.

(1) $gcd\big(f(v), f(v_{2i-1})\big) = \begin{cases} gcd(1,5i-2) & ;\ i \text{ is odd} \\ gcd(1,5i-3) & ;\ i \text{ is even} \end{cases}$

$$= 1$$

$$gcd\big(f(v), f(vv_{2i-1})\big) = \begin{cases} gcd(1,5i-3) & ;\ i \text{ is odd} \\ gcd(1,5i-2) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

and $gcd\big(f(v_{2i-1}), f(vv_{2i-1})\big) = \begin{cases} gcd(5i-2,5i-3) & ;\ i \text{ is odd} \\ gcd(5i-3,5i-2) & ;\ i \text{ is even} \end{cases}$

$$= 1$$

(2) $gcd\big(f(v), f(v_{2i})\big) = \begin{cases} gcd(1,5i) & ;\ i \text{ is odd} \\ gcd(1,5i+1) & ;\ i \text{ is even} \end{cases}$

$$= 1$$

$$gcd\big(f(v), f(vv_{2i})\big) = \begin{cases} gcd(1,5i+1) & ;\ i \text{ is odd} \\ gcd(1,5i) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

and $gcd\big(f(v_{2i}), f(vv_{2i})\big) = \begin{cases} gcd(5i,5i+1) & ;\ i \text{ is odd} \\ gcd(5i+1,5i) & ;\ i \text{ is even} \end{cases}$

$$= 1$$

(3) $gcd\big(f(v_{2i-1}), f(v_{2i})\big) = \begin{cases} gcd(5i-2,5i) & ;\ i \text{ is odd} \\ gcd(5i-3,5i+1) & ;\ i \text{ is even} \end{cases}$

$$= 1$$

$$gcd\big(f(v_{2i-1}), f(v_{2i-1}v_{2i})\big) = \begin{cases} gcd(5i-2,5i-1) & ;\ i \text{ is odd} \\ gcd(5i+1,5i-1) & ;\ i \text{ is even} \end{cases}$$

$$= 1$$

and $gcd\big(f(v_{2i}), f(v_{2i-1}v_{2i})\big) = \begin{cases} gcd(5i,5i-1) & ;\ i \text{ is odd} \\ gcd(5i+1,5i-1) & ;\ i \text{ is even} \end{cases}$

$$= 1$$

Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.

Hence Friendship graphF_n is an edge vertex prime.

Illustration 2.4: Edge vertex prime labeling forF_{15}.

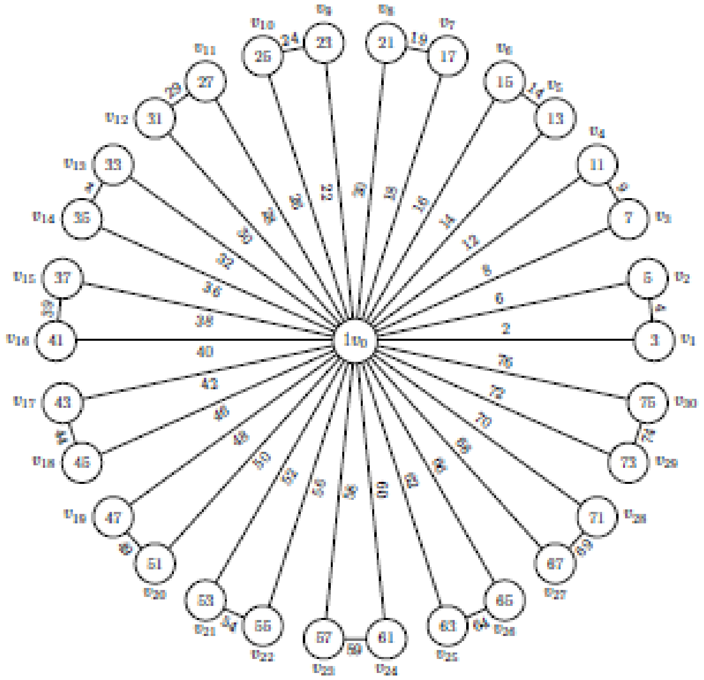

Fig. 2.4F_{15}

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