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An Alternative Robust Test of Lagrange Multiplier for ARCH Effect

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ABSTRACT:*ARCH model has become very popular in that it enables the econometrician to estimate the variance of a series at a particular point in time. The aim of my paper is as follows: to study the tests of ARCH model in time series data, to study the effect of outliers in ARCH model, to study the test of ARCH model in time series data after using robust method and to study the test effect of ARCH model in time series data by simulation. In this paper, I propose a new robust t-test of ARCH. The usefulness of the proposed method is investigated by some well-known data sets as well as Monte Carlo simulation studies.*

KEYWORDS: *Time Series; Homoscedasticity; ARCH; Outliers; LM test; t-test of ARCH; Monte Carlo simulation*

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I. Introduction

Usual knowledge, heteroscedasticity is a warning of cross-sectional data and autocorrelation is a notice of time series data. In developing the advance time dependent data analysis, time series are often plagued by the problem of heteroscedasticity. Modern analysis of financial data and econometric data, such as inflation rates, exchange rates, volatility of market return, foreign exchange market as well as real compensation-productivity data etc, have found rich confirmation of clustering large and small turbulence, which imply a form of heteroscedasticity in which the variance of the disturbance depends on the size of the preceding disturbance, autoregressive conditional heteroscedasticity, or ARCH model as a choice to the typical time series treatment [1]. If the value of any volatility term in any particular period is correlated with its own preceding value, then we say that ARCH is present in the variables. An ARCH model considers the variance of the current error term to be a function of the variables of the previous time periods volatility terms [2,3]. In other words, there is a particular kind of heteroscedasticity present in which the variance of the regression volatility depends on the volatility in the past [4]. There are several reasons for ARCH one of them is when the data set contains very large and small values or there is evidence of a “clumping” of a large and small volatility [5].

There are two formal graphical tests for detection of ARCH, which are dependent variable against index (data type, such as month, year, day and etc.) as well as residuals against index [2, 4, 6, 8]. The graphical methods are very easy to understand when there is a clear-out picture. But it may often produce ambiguous pictures and analysts may come up with conflicting conclusions. That is why more formal test like the analytical test is required. In our reviews, books index [2, 4, 6, 7, 8] and journal articles [1, 5, 9, 10, 11, 12]the most celebrated test for the detection of ARCH is Lagrange Multiplier (LM) test [1]. This test is suitable to detect the order of ARCH. Although extremely popular, the LM test is not always an appropriate test for detection of ARCH. This test suffers a huge set back in the presence of outliers or outlying observations. For that reason we propose a new robust t-test to detect the order of ARCH when data set contains outliers or free from outliers. Section 2 presents ARCH model. Proposed t-test of ARCH is presented in section 3. Examine the ARCH model in section 4. Section 5 presents Monte Carlo simulation study.

II. ARCH Model

Lots of economic time series display periods of volatility. Conditionally heteroscedasticity models tolerate the conditional variance of a series to depend on the past understanding of the error process. A large comprehension of the current period’s disturbance increases the conditional variance in successive periods. For a steady process, the conditional variance will eventually decompose to the long-run variance. As such, ARCH model can detain periods of turbulence and tranquility.
Mathematically, a modern version of the k-variable regression model can be expressed in the following format

$$Y_t = \beta_1 + \beta_2 X_{2t} + \cdots + \beta_k X_{kt} + \varepsilon_t$$

Or

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_{k-1} Y_{t-k+1} + \varepsilon_t \, , \; t = 1, 2, \cdots, T$$

To use the conditional information, in general, we have to assume that

$$\varepsilon_t \sim N\left(0, \left(\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2\right)\right)$$

Now, we conclude our discussion of ARCH model by considering the aforesaid model, which takes the following form

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + v_t \ldots\ldots (1)$$

This is the ARCH (p) Model.

Considering an autoregressive process with Gaussian noise in equation (1), [13] at first consider outliers within time series. There are two type of outliers are defined mostly: 'Additive Outliers (AO)' where a single observation is affected and 'Innovation Outliers (IO)' where an unusual observation in the generating process affects all later observations and the subsequent series [13]. In this paper, we suggest that innovations or disturbances based on the ordinary least square (OLS) fit of the model should be more convincing and reliable in a diagnostic test for detection of ARCH effect of the disturbance in time series data.

III. Propose Test

Conditional variance is a measure of risk. ARCH effect has been included in a regression framework to test the hypothesis of risk-averse agents. Consider the following different order ARCH model-

ARCH (1): $\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + v_t$

ARCH (2): $\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + v_t$

$\vdots$

ARCH (p): $\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + v_t$

To see if these are the case, we develop the following model

$$| \hat{v}_t | = \gamma_0 + \gamma_1 \hat{\varepsilon}_t^2 + \xi_t$$

Where $\hat{v}_t$ and $\hat{\varepsilon}_t^2$ are the estimated values of v_t and ε_t^2 respectively.

To test the hypothesis

$H_0 : \gamma_1 = 0, i.e.,$ There is no ARCH effect

Versus

$H_1 : \gamma_1 \neq 0, i.e.,$ There is an ARCH effect

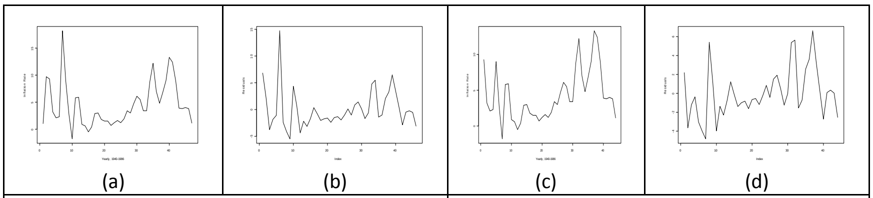
Now, we would like to test the null hypothesis, the usual test statistic of t can be written as

Where, $se(\hat{\gamma}_1) = \sqrt{\sum \left(\left(\hat{\varepsilon}_t^2 - mean\left(\hat{\varepsilon}_t^2\right)\right)^2 \hat{v}_t^2\right) \Big/ \left(\sum \left(\hat{\varepsilon}_t^2 - mean\left(\hat{\varepsilon}_t^2\right)\right)^2\right)^2}$ refers to the estimated standard error and $\hat{v}_t^2$ is the estimated squared errors of tested ARCH model. It can be shown that the t variable thus defined follows the t distribution with degrees of freedom and T is the number of observations.

IV. Results

4.1. Real Data Examples

In the previous section, we describe theoretical background of ARCH. Now, in this section, apply some formal graphs and test to detect ARCH. Also apply our newly propose robust t-test to detect ARCH. At first, run the time series by the OLS to obtain the residuals. Then plot the data with two types and also apply analytical tests in the data sets. Let us first consider the USA inflation rate data [3]. Our second data set is US/UK exchange rate [2]. Our third data set is price of crude oil on the dollar rate, which has taken from [4]. In their books, [3] considered LM test of ARCH as the only analytical test, alternatively, [2] as well as [4] considered graphical aforementioned two tests and LM test of ARCH for detection of ARCH effect in the data sets. On the basis of these data sets the test results of the classical suggested tests and our newly proposed tests are given in the following below-



Aforesaid figures present the data with outliers | Above-mentioned figures express the data without outliers

Figure 1. Graphical displays of USA inflation rate data

From Fig. 1 (a-d) the variation in the series appears to be fluctuating, with several clusters of large and small movements for both situations. According to the definition of graphical ARCH test, there is the strong appearance of heteroscedasticity.

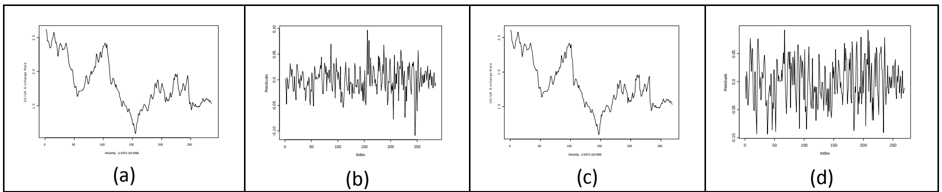
Table 1: ARCH Tests performance of USA Inflation Rate data with and without Outliers

	Statistic Value		***Critical Value (5%)***		***P-Value***	
	With Outliers	Without Outliers	With Outliers	Without Outliers	With Outliers	Without Outliers
LM(1)	0.04662608	**5.361923**	3.841459	3.841459	0.8290417	0.02058106
Propose t-Test(1)	**1.722152**	**1.733745**	1.681071	1.682878	0.04603281	0.04523917

Table 1 shows that LM test fail to detect the existence of ARCH when data set contains outlying observations. Alternatively, when the data set free from outliers, it detect the existence of ARCH (1). But, our newly propose t-test detect the presence of ARCH (1) when the data set hold outliers or free from outliers. There is no ARCH effect in the data set [3].

4.2. US/UK Exchange Rate Data

This data set holds 286 observations, at first we make sure the outliers by the robust LTS method [14]; it identify 16 outliers (case 3, 48, 87, 97, 106, 110, 111, 156, 159, 187, 205, 219, 227, 245, 246 and 247). Then apply prescribed graphical tests, classical LM test of ARCH as well as our newly proposed t-test of ARCH before and after erasing outliers, which outcome have publicized below-



On top of figures present the data with outliers | Afore-mentioned figures express the data without outliers

Figure 2. Graphical exhibits of US/UK exchange rate data

From Fig. 2 (a) & (c) show the pattern are considerably ups and downs fluctuations over the sample period. In Fig. 2 (b) & (d) denote the periods of wide swings for some time period and the periods of rather moderate swings in other time periods. According to the definition of graphical ARCH tests, the plots exhibit considerably volatility clustering.

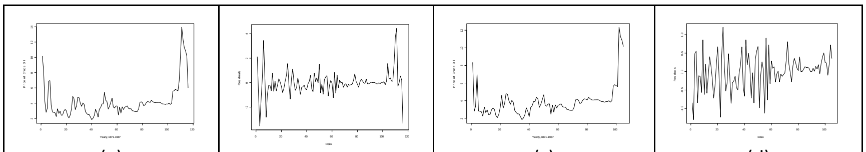
Table 2: ARCH Tests performance of US/UK Exchange Rate data with and without Outliers

	Statistic Value		***Critical Value (5%)***		***P-Value***	
	With Outliers	Without Outliers	With Outliers	Without Outliers	With Outliers	Without Outliers
LM(1)	8.1508	**0.4821653**	3.841459	3.841459	0.004304217	0.4874432
Propose t-Test(1)	**1.602978**	**0.3196948**	1.650237	1.650602	0.05502947	0.3747253

Outcome presented in Table 2 shows that the LM test fails to declare the correct conclusion of the data set when outliers present in the data. On the other hand, when the data set free from outlying observations, it declare that there is no ARCH effect in the data. But, our newly propose t-test announce the right finding that there is no ARCH effect when the data set contain outliers and free from outliers. It is worth-mention that there is strong indication of ARCH (1) [2].

4.3. Price of Crude Oil on Dollar Rate Data

This data set has 117 observations, at first we ensure the outliers by the robust LTS method [14]; it finds out 11 outliers (case 1, 3, 6, 8, 25, 50, 104, 109, 110, 111 and 116). Then apply recognized graphical tests, classical LM test of ARCH as well as our newly proposed t-test of ARCH before and after removing outliers, which results have shown below-



(a) (b) (c) (d)

Above figures present the data with outliers | Above-mentioned figures express the data without outliers

Figure 3. Graphical exhibits of price of crude oil on the dollar rate data

From Fig. 3 illustrate the behavior of the price of crude oil on dollar rate in macroeconomic analysis. From casual inspection, Fig. 3 (a) & (c) show the trends to smooth out long-term fluctuations. In Fig. 3(b) & (d) show the pattern tranquil alongside periods with large increase and decrease of volatility. According to the definition of graphical ARCH tests, such series display noticeably conditionally heteroscedastic.

Results presented in Table 3 shows that LM test fail to identify the exact inference when the data set holds outliers. Conversely, when the data free from outliers, it declare there is no ARCH effect in the data. But, when the data set holds outliers and free from outliers, our newly propose t-test declare that there is no ARCH effect in the data set. It is to be noticeably that there is strong evidence of ARCH (1) [4].

Table 3: ARCH Tests performance of Price of Crude Oil on the Dollar Rate Data with and without Outliers

	Statistic Value		***Critical Value (5%)***		***P-Value***	
	With Outliers	Without Outliers	With Outliers	Without Outliers	With Outliers	Without Outliers
LM(1)	9.370025	**1.911362**	3.841459	3.841459	0.002205625	0.166812
Propose t-Test(1)	**1.553098**	**0.9758428**	1.65845	1.65993	0.06159806	0.1657255

To sum up the aforesaid discussion we have demonstrated that for USA inflation rate, US/UK exchange rate and price of crude oil on the dollar rate data sets give opposite outcome when outliers present in the data sets. We study that the LM test is very sensitive to outliers. Overall, we may conclude that our newly propose t-test is perfect to detect the order of ARCH when outlying observations present in the data set or absent.

V. Simulation Study

In the preceding section, we consider few real data sets to see how the outliers affect the detection procedure of ARCH effect in a time series data. But examples are not enough to get a final answer to a problem. In this section, we report a Monte Carlo simulation that is design to compare the power performance of the LM test of ARCH as well as newly proposed t-test of ARCH. In order to compare the power performance of aforesaid tests, we simulate artificial data sets in the following way: to simulate homoscedastic and ARCH data independently from uniform distribution based on time series in five different sample sizes, i.e. T= 50, 100, 200, 500 and 1000 respectively. Each experiment is run 10,000 times and the test power of the both cases are given in Table 4.

Table 4. Simulation Power of ARCH tests under Homoscedasticity ($\alpha = 0.05$)

	Power(in percentage)				
	T-50	T=100	T=200	T=500	T=1000
LM(1)	11.41	13.54	18.88	27.79	34.64
Propose t-Test(1)	**0.89**	**0.45**	**0.29**	**0.24**	**0.09**

To investigate the power of these two tests under homoscedasticity, the LM test, the rejection power of null hypothesis (H_0) is very much higher than our newly proposed t-test when null hypothesis (H_0) is true. Alternatively, our newly propose t-test, the rejection power of null hypothesis (H_0) is very low than LM test when null hypothesis (H_0) is true when the data set.

Table 5: Simulation Power of ARCH tests under ARCH Errors (H_0)

	Power (in percentage)				
	T-50	T=100	T=200	T=500	T=1000
LM(1)	9.30	13.89	17.49	24.57	29.39
LM(2)	16.40	19.51	23.37	26.89	31.19
LM(3)	17.39	21.27	24.13	28.79	34.41
LM(4)	21.14	23.98	27.69	33.54	39.59
Propose t-Test(1)	**99.94**	**100**	**100**	**100**	**100**
Propose t-Test(2)	**99.95**	**100**	**100**	**100**	**100**
Propose t-Test(3)	**99.98**	**100**	**100**	**100**	**100**
Propose t-Test(4)	**99.99**	**100**	**100**	**100**	**100**

To examine the simulated power of these two tests under ARCH, from Table 5 shows that our newly propose t-test, the rejection power of null hypothesis (H_0) is much higher than LM test when alternative hypothesis (H_0) is true. Conversely, the performance of LM test is very poor and it power is less than 50% for this simulation.

We demonstrate that the power of the newly proposed t-test of ARCH is very high than LM test of ARCH for homoscedastic and heteroscedastic situation. Notice that the rejection power of null hypothesis (H_0) of this test is much higher than LM test of ARCH when alternative hypothesis is true in different sample sizes. Therefore, we can say that the newly proposed t-test is appropriate than any other tests to detect the order of ARCH.

VI. Conclusion

ARCH is potentially a serious problem specific to time series data. If the volatility is ARCH in a linear model, the OLS estimator will be consistent but inefficient under the usual assumptions. Hence, the tests of hypothesis become invalid and give seriously misleading conclusions about the statistical significance of the estimated regression coefficients as well as the model. For these consequences, it is necessary to detect the existence of ARCH in a given series of data. There are different types of tests for the incidence of ARCH. We can apply these tests to detect the presence of ARCH. But these existing tests have serious drawback. The most popular and frequently used test is Lagrange Multiplier (LM) test. This test can correctly identify the order of ARCH. But this test is very much sensitive to outlying observations. Consequently, we develop new robust test of ARCH, which is t-test of ARCH namely and it detect the order of ARCH correctly. We have seen that irrespective of the presence of outliers or not, the newly proposed t-test of ARCH performs much better than other tests for different sample sizes.

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