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A Swine Flu Model with Mixed Transmission and Disease Induced Death Rate

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Abstract *In this paper we analyzed an SEIR compartment model of a Swine flu with mixing transmission and disease induced death rate. The stability of the diseases-free equilibrium and the endemic equilibrium is obtained by Routh-Hurwitz criteria. The basic reproduction number* R_0 *has also been discussed, when* $R_0 < 1$ *, the disease free equilibrium point is stable. In case* $R_0 > 1$ *, there exists endemic equilibrium. Numerical simulation is carried out for different values of contact rate to understand the transmission behavior of the disease.*

Keywords: *Mathematical Epidemic Model, Equilibrium Point, Endemic Point, Stability Anal*ysis.

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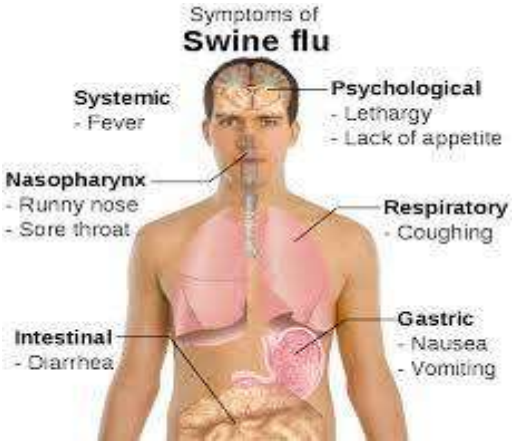
I. INTRODUCTION

Mathematical model have become important tools to study and analyze the spread and control of infectious diseases. Almost all mathematical models of diseases start from the same basic premise; that the population can be subdivided into a set of distinct classes, dependent upon their experience with respect to the diseases. In this disciplinary V. H. Badshah and Amit Kumar [20] gave a primary result of mathematical modeling. Cappaso and Serio [19] introduced a saturated incidence rate into epidemic model. Mena – Lorca and Hethcote [5] also analyzed an SIRS model with the same saturation incidence. Ruan and Wang [16] studied an epidemic model with a specific nonlinear incident rate. Liu et al. [22,23], Derrick and Ven den Driessche[24] Hethcote and Ven Den Driessche [4] proposed an epidemic model with non-monotonic incidence rate. After that Xiao and Raun [27] discussed non-monotonic incidence rate.

Swine Flu is highly communicable respiratory disease "refers to infection caused by those strains of infection virus, called the swine influenza virus, that generally infect pigs". Symptoms of swine flu are fever, which is usually high; cough; runny nose or stuffy nose; sore throat; body aches; headache; chills; fatigue or tiredness, which can be extreme; diarrhea and vomiting, sometimes, but more commonly seen than with seasonal flu.



Several different incidence rates have been planned by many researchers [see for instance, Anderson and May [14], Elsteva and Matias [6], Hethcote and Driesech [4], Ruan and Wang [16], Liu, et. al. [22,23] Derrick and Ven den Driessche [24] , Alexander and Moghadas [7] and Xiao and Raun [27] , in year 2010 Pathak et. al. gave a result on rich dynamics of an SIR epidemic model Recently Porwal, et. al. [9,10,11,12]] presented their work on respected field .

II. THE MATHEMATICAL MODEL

2.1 Basic Model

Nidhi et. al. [8] has proposed an SEIR epidemic model with mixed transmission which describes swine flu effect of certain serious diseases on the community when the number of infective is getting larger. Under by differential equations

$$\frac{dS}{dt} = A - \frac{cSI}{S+I} + rI - \mu S, \tag{1}$$

$$\frac{dE}{dt} = \frac{cSI}{S+I} - (\lambda + \mu)E, \tag{2}$$

$$\frac{dI}{dt} = \lambda E - (r + \gamma + \mu +)I, \tag{3}$$

$$\frac{dR}{dt} = \gamma I - \mu R, \tag{4}$$

where $S(t)$, $I(t)$, $R(t)$ represent the number of susceptible, infective and recovered individual respectively.

A = the recruitment rate of the population,

μ = the natural death rate of the population,

c = the contact rate at which the susceptible population is converted into exposed population. rate of the population,

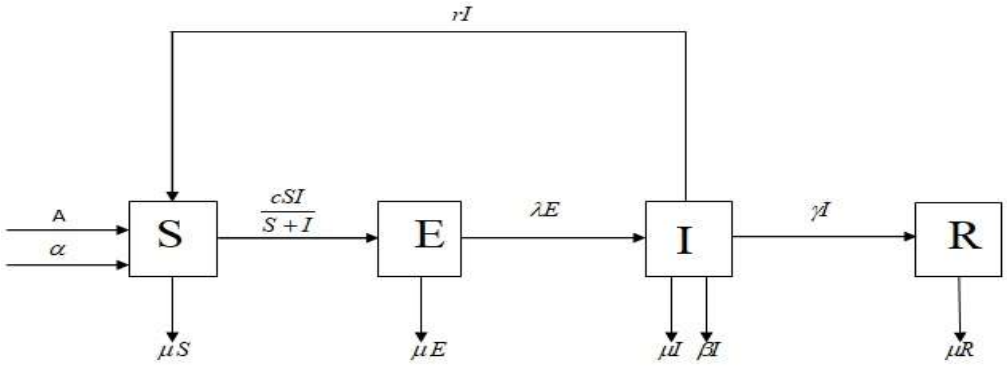
r = the recovery rate,

λ = the effective transmission coefficient,

γ = the natural recovery rate of the infective individuals.

2.2. Model for Induced death rate and Immigration rate.

Now the transmissions of shown as figure.



Now the model (2.2) with disease induced death rate β, and immigration rate α, shown as :

$$\frac{dS}{dt} = A - \frac{cSI}{S+I} + rI - \mu S + \alpha, \tag{5}$$

$$\frac{dE}{dt} = \frac{cSI}{S+I} - (\lambda + \mu)E, \tag{6}$$

$$\frac{dI}{dt} = \lambda E - (r + \gamma + \mu + \beta)I, \tag{7}$$

$$\frac{dR}{dt} = \gamma I - \mu R, \tag{8}$$

where

$S(t)$ = the number of susceptible,

$I(t)$ = the number of infective,

$R(t)$ = the number of recovered person respectively.

A = the recruitment rate of the population,

μ = the natural death rate of the population,

c = the contact rate at which the susceptible population is converted into exposed population,

r = the recovery rate,

λ = the effective transmission coefficient,

γ = the natural recovery rate of the infective individuals,

β = is the induced death rate,

α = is the immigration rate.

III. MATHEMATICAL ANALYSIS

SIR MODEL WITH EQUILIBRIUM POINT AND STABILITY ANALYSIS

In this case the system equation (5) to (8) reduce to the equilibrium points the above differential equation should be equated to zero.

$$A - \frac{cSI}{S+I} + rI - \mu S + \alpha = 0, \tag{9}$$

$$\frac{cSI}{S+I} - (\lambda + \mu)E = 0, \tag{10}$$

$$\lambda E - (r + \gamma + \mu + \beta)I = 0, \tag{11}$$

$$\gamma I - \mu R = 0, \tag{12}$$

When $I = 0$, the equation (9) gives

$$A - \frac{cSI}{S+I} + rI - \mu S + \alpha = 0,$$

$$S = \frac{A+\alpha}{\mu}.$$

By equation (12) we have,

$$\gamma I - \mu R = 0,$$

$$R = \frac{\gamma I}{\mu}, \text{ Gives } R = 0.$$

Again by equation (11)

$$\lambda E - (r + \gamma + \mu + \beta)I = 0,$$

$$E = \frac{(r + \gamma + \mu + \beta)}{\lambda} I, \text{ Gives } E = 0.$$

The system of equation (9) to (12) always has the DFE $P_0 = \left(A + \alpha \big/ \mu, 0, 0, 0 \right)$ for any set of parameter values. The given system has unique endemic equilibrium point $P^* = (S^*, E^*, I^*, R^*)$ we write from equation (11)

$$E = \frac{(r + \gamma + \mu + \beta)}{\lambda} I, \tag{13}$$

Therefore substituting equation (13) by (10) we get,

$$\frac{cSI}{S+I} - (\lambda + \mu)E = 0,$$

$$\frac{cS}{S+I} = p,$$

$$cS = pS + pI, \tag{14}$$

$$S = \frac{p}{c-p} I. \tag{15}$$

Now we substituting (14) in (9) take $p = \frac{cS}{S+I}$. in equation (9)

$$A - pI + rI - \mu S + \alpha = 0,$$

Now finally put $S = \frac{p}{c-p} I$, in equation,

$$A - pI + rI - \mu \frac{p}{c-p} I + \alpha = 0,$$

$$I^* = \frac{A+\alpha}{\left[\frac{\mu p}{c-p} + p - r\right]}, \tag{16}$$

By equation (12), now

$$\gamma I - \mu R = 0,$$

$$R = \frac{\gamma}{\mu} I, \tag{17}$$

Then the endemic equilibrium point $P^* = (S^*, E^*, I^*, R^*)$ is

$$S^* = \frac{p}{c-p} I^*, \tag{18}$$

$$E^* = \frac{(r+\gamma+\mu+\beta)}{\lambda} I^*, \tag{19}$$

$$I^* = \frac{A}{\left[\frac{\mu p}{c-p} + p - r\right]}, \tag{20}$$

$$R^* = \frac{\gamma}{\mu} I^*, \tag{21}$$

Where $p = \frac{(\lambda+\mu)(r+\gamma+\beta+\mu)}{\lambda}$, (22)

The basic reproduction number $R_0 = \frac{c\lambda}{(\lambda+\mu)(r+\gamma+\beta+\mu)}$. (23)

Theorem 3.1. The disease free equilibrium of the system is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof. We consider equations

$$F_1 = A - \frac{cSI}{S+I} + rI - \mu S + \alpha, \tag{24}$$

$$F_2 = \frac{cSI}{S+I} - (\lambda + \mu)E, \tag{25}$$

$$F_3 = \lambda E - (r + \gamma + \mu + \beta)I, \tag{26}$$

$$F_4 = \gamma I - \mu R, \tag{27}$$

Now applying Jacobian matrix

$$J_0 = \begin{bmatrix} \frac{-cI^2}{(S+I)} & 0 & \frac{-cS^2}{(S+I)^2} + r & 0 \\ \frac{cI^2}{(S+I)} & -(\lambda+\mu) & \frac{cS^2}{(S+I)^2} & 0 \\ 0 & \lambda & -(r+\gamma+\mu+\beta) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}.$$

At equilibrium point $P_0 = \left(A + \alpha/\mu, 0, 0, 0\right)$ the Jacobian matrix becomes

$$J_0 = \begin{bmatrix} -\mu & 0 & -c + r & 0 \\ 0 & -(\lambda+\mu) & c & 0 \\ 0 & \lambda & -(r+\gamma+\mu+\beta) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}.$$

The characteristics equation $\left|J_0 - \phi I\right| = 0$, is given as

$$\begin{vmatrix} -\mu-\phi & 0 & -c+r & 0 \\ 0 & -(\lambda+\mu)-\phi & c & 0 \\ 0 & \lambda & -(r+\gamma+\mu+\beta)-\phi & 0 \\ 0 & 0 & \gamma & -\mu-\phi \end{vmatrix} = 0,$$

$$-(\mu+\phi)\begin{vmatrix} 0 & 0 & -(\mu+\phi) \\ c & -(r+\gamma+\mu+\beta+\phi) & \gamma \\ -(\lambda+\mu+\phi) & \lambda & 0 \end{vmatrix} = 0,$$

$$-(\mu+\phi)\left[0 - 0 - (\mu+\phi)\begin{vmatrix} c & -(r+\gamma+\mu+\beta+\phi) \\ -(\lambda+\mu+\phi) & \lambda \end{vmatrix}\right] = 0,$$

$$(\mu+\phi)^2\left[(r+\gamma+\mu+\beta+\phi)(\lambda+\mu+\phi) - c\lambda\right] = 0,$$

$$(\mu+\phi)^2\left[(\lambda+\mu+\phi)(r+\gamma+\mu+\beta+\phi) - c\lambda\right] = 0, \tag{28}$$

It is clear that two eigen values $\phi = -\mu, -\mu$ are negative, other eigen values are given by the quadratic equation

$$\phi^2 + a_1\phi + a_2 = 0.$$

Therefore, by Routh- Huritz criteria the disease- free equilibrium stable if $a_1 > 0$ *and* $a_2 > 0$ and $(\lambda+\mu)(r+\gamma+\beta+\mu) > c\lambda$, i.e. $R_0 < 1$.

Theorem 3.2. If $R_0 > 1$ the endemic equilibrium P^* is locally asymptotically stable.

Proof. The variation matrix at the endemic point $P^* = (S^*, E^*, I^*, R^*)$

$$J_1 = \begin{bmatrix} \dfrac{-cI^{*2}-\mu}{\left(S^*+I^*\right)^2} & 0 & \dfrac{-cS^{*2}}{\left(S^*+I^*\right)^2}+r & 0 \\ \dfrac{cI^{*2}}{\left(S^*+I^*\right)} & -(\lambda+\mu) & \dfrac{cS^{*2}}{\left(S^*+I^*\right)^2} & 0 \\ 0 & \lambda & -(r+\gamma+\mu+\beta) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

Consider that

$$z_1 = \frac{cI^{*2}}{\left(S^*+I^*\right)^2} \quad \text{and} \quad z_2 = \frac{cS^{*2}}{\left(S^*+I^*\right)^2}.$$

Then J_1 becomes

$$J_1 = \begin{bmatrix} -z_1-\mu & 0 & -z_2+r & 0 \\ z_1 & -(\lambda+\mu) & z_2 & 0 \\ 0 & \lambda & -(r+\gamma+\mu+\beta) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}.$$

The characteristics equation $\left|J_1-\phi I\right|=0$, is given as

$$\begin{vmatrix} -z_1-\mu-\phi & 0 & -z_2+r & 0 \\ z_1 & -(\lambda+\mu)-\phi & z_2 & 0 \\ 0 & \lambda & -(r+\gamma+\mu+\beta)-\phi & 0 \\ 0 & 0 & \gamma & -\mu-\phi \end{vmatrix} = 0,$$

$$-(\mu+\phi)\begin{bmatrix} -z_2+r(0-z_1\lambda)-z_2[0+\lambda(z_1+\mu+\phi)]-(r+\gamma+\mu+\beta+\phi)] \\ [0+(\lambda+\mu+\phi)(z_1+\mu+\phi)] \end{bmatrix} = 0,$$

$$(\mu+\phi)\begin{bmatrix} (z_1+\mu+\phi)(\lambda+\mu+\phi)(r+\gamma+\mu+\beta+\phi) \\ -(z_1+\mu+\phi)z_2\lambda+(z_2-r)z_1\lambda \end{bmatrix} = 0, \quad (29)$$

It is clear that one eigen value is negative $\phi=-\mu$ and other eigen values are given by the cubic equation.

$$\phi^3+a_1\phi^2+a_2\phi+a_3=0,$$

Where he cubic equation

$$a_1 = 3\mu+\lambda+z_1+r+\gamma=, \quad (30)$$

$$a_2 = [(\lambda+\mu)(z_1+\mu)+(z_1+2\mu+\lambda)(r+\gamma+\beta+\mu)-z_2\lambda], \quad (31)$$

$$a_2 = (\lambda+\mu)(z_1+\mu)(r+\gamma+\beta+\mu)-(\mu z_2+rz_1)\lambda, \quad (32)$$

By Routh-Hurwitz criteria, the system (2.2) is locally asymptotically stable if $a_1>0$, $a_3>0$ and $a_1a_2>a_3$. Thus, P^* is locally asymptotically stable.

IV. NUMERICAL ANALYSIS.

1.1. FOR DISEASE FREE EQUILIBRIUM.

From the numerical values of the parameters as $A=1,\ c=0.3,\ r=0.1,\ \mu=0.02,\ \lambda=0.1,\ \gamma=0.01,\ \beta=0.02$, and $\alpha=0.2$, Then the calculated disease free equilibrium point and basic reproductive number are: $P_0(S,0,0,0)=(60,0,0,0)$, and $R_0=0.01666<1$. These values shows that $S(t)$, goes to its steady state, while $E(t)$, $I(t)$, and $R(t)$, goes to zero with respect to time. Hence the disease dies out.

1.2. FOR ENDEMIC EQUILIBRIUM

We change the value of $c = 0.003$, and all other parameters are as above. Then we obtain $P^*(S^*, E^*, I^*, R^*) = (16.3635, 16.3635, 10.909, 5.4545)$ and $R_0 = 0.16666 > 1.$ therefore, the endemic equilibrium P^* is locally asymptotically stable. These values shows that number of susceptible population are also same number of exposed population then disease S, E, I and R, goes to their steady state values. Hence the disease becomes endemic.

V. CONCLUSION

In this paper we have carried out result on SEIR model of Swine Flu model with mixed transmission and induced death rate and the existence of stability of disease- free and endemic equilibrium. In this model we used Routh–Hurwitz Stability Criterion to provide the critical evaluation of the epidemics of Swine Flu, and apply Jacobian method to determine the effect of variations in the potential of the epidemic for their prediction. Stability of infectious disease through the model is depending upon eigen value and create the given system slightly stable.

Where the basic reproduction number $R_0 = \frac{c\lambda}{(\lambda + \mu)(r + \gamma + \beta + \mu)}$. Our main result indicates that when $R_0 < 1$, the diseases-free equilibrium is P_0 is stable and when $R_0 > 1$, the endemic equilibrium P^* exist and locally asymptotically stable. For verify our result we used numerical analysis for verification of our result.

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