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Monte Carlo Simulation with Variance Reduction Methods for Chained Option

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Abstract : *Monte Carlo simulation with the variance reduction methods is generally more efficient than general Monte Carlo simulation because variance reduction method causes small standard deviation. In this paper, we compare two simulation methods for the barrier option called chained option and confirmed that Monte Carlo simulation used the variance reduction method is also more efficient than general Monte Carlo simulation.*
Keywords:*Barrier option, Monte Carlo simulation*

I. Introduction

Barrier options are a widely used class of path-dependent financial derivative securities and path-dependent exotic options that are similar in some ways to ordinary options. Merton[1]has derived a down-and-out call price by solving the corresponding partial differential equation with some boundary conditions. ReinerandRubinstein[2] published closed form pricing formulas for various types of single barrier options. Rich[3] provided a mathematical framework to value barrier options. In these papers, the underlying asset price is monitored with respect to a single constant barrier for the entire life of the option. Many authors have studied more complicated structures of barrier options because of their popularity in a market. These are double barrier option with two barriers. Kunitomo and Ikeda[4] derived a pricing formula for double barrier options with curved boundaries as the sum of an infinite series. Geman and Yor[5]provided a probabilistic approach to derive the Laplace transform of the double barrier option price. Pelsser[6] inverted the Laplace transforms of the probability density functions using contour integration and derived analytical formulas for pricing a variety of double barrier options.

All of the above mentioned papers are about barrier options where monitoring of the barrier begins at a predefined time.There is another class in barrier options with two barriers.Another barrier option is executed when the main barrier is reached. These options are well known in the over-the-counter and currency derivatives markets.Jun and Ku[7] derived closed form valuation formulas for chained barrier options of various type with constant barriers by applying the reflection principle and Girsanov's Theorem. And Jun and Ku[8] extended to the cases that the barriers are exponential functions and general curved functions.

Monte Carlo methods are often used when other methods are difficult to implement due to the complexity of the problem. The disadvantage of Monte Carlo methods can be computationally burdensome to achieve a high level of accuracy. To reduce the computational burden of standard Monte Carlo methods a variety of variance reduction methods([9],[10]) have been proposed. This paper usesa variance reduction method which is called Antithetic Variate method for pricing the chained option.

This paper is organized as follows. In section II, the concepts of barrier option and chained option are introduced and their option pricing formulas are explained. Section III shows the numerical results of two Monte Carlo simulations for the chained option. Concluding remarks are given in section IV.

II. Barrier Option And Chained Option

Barrier options are a widely used class of path-dependent derivative securities. Barrier options are the same as plain vanilla options except for the fact that it becomes active only after underlying asset crosses a certain price, known as the barrier. For example, an up-and-in call option (UIC) gives the option holder the payoff of a call if the price of the underlying asset reaches a higher barrier level during the option's life, and it pays off zero unless the asset price reaches that level. For an up-and-out call (UOC), the option becomes worthless if the underlying asset price hits a higher barrier, and its payoff at expiration is a call otherwise. Options with a lower barrier level are said to be down-and-in and down-and-out options.

Let r be the risk-free interest rate and σ be a constant. Assume the price of the underlying asset S follows a geometric Brownian motion

$$S_t = S_0 e^{\Box t+\sigma W_t}$$

where $\bar{\mu} = r - \frac{\sigma^2}{2}$ and W_t is a standard Brownian motion under the risk-neutral probability.

Consider a European call expiring at T with strike price K, up-barrier $U(> S_0)$, and down-barrier $D(< S_0)$. Then the up-and-in call option value[11] is

$$UIC = S_0\left[N(d_1) + \left(\frac{U}{S_0}\right)^{\frac{2\tilde{\mu}}{\sigma^2}} N(d_2) - \left(\frac{U}{S_0}\right)^{\frac{2\tilde{\mu}}{\sigma^2}} N(d_3)\right]$$
$$- e^{-rT}K\left[N(d_1 + \sigma\sqrt{T}) + \left(\frac{U}{S_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} N(d_2 - \sigma\sqrt{T}) - \left(\frac{U}{S_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} N(d_3 + \sigma\sqrt{T})\right].$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{S_0}{U}\right) + \frac{\bar{\mu}}{\sigma}\sqrt{T}, \quad d_2 = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{S_0}{U}\right) - \frac{\bar{\mu}}{\sigma}\sqrt{T}, \quad d_3 = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{S_0 K}{U^2}\right) - \frac{\tilde{\mu}}{\sigma}\sqrt{T},$$

$\tilde{\mu} = r + \frac{\sigma^2}{2}$, and $N(x)$ is the cumulative standard normal distribution function.

The up-and-out call option (UOC) can get from knock-in knock-out relation.

$UOC = \text{Vanilla Call} - UIC$

People would buy barrier options, because it carries a much lower extrinsicvalue than plain vanilla options. If one expects the underlying stock to rallystrongly, an up-and-in barrier option can result in a higher profitability thanplain vanilla option simply it is cheaper. The advantages of barrier options arethey are cheaper than conventional options, resulting in higher profit shouldbarrier criteria is satisfied. The disadvantage is higher risk of loss due to barrierfeatures.

Chained option has also two barriers. For chained options, a regular barrier option is activated when a primary barrier is hit. For example, a down-and-in chained call (DIC_u) is a down-and-in call option is activated at time when the underlying asset price hits an upper barrier level.

The valuation formula for a down-and-in call option commencing at time when the asset price hits the up-barrier U are as follows (see [7] for more detail).

First, suppose $K > D$. The knock-in call option value at time 0, DIC_u, which is activated at time $\tau_u = \min\{t: S_t = U, U > S_0\}$ is

$$DIC_u = S_0\left(\frac{D}{U}\right)^{\frac{2\tilde{\mu}}{\sigma^2}} N(z_1) - e^{-rT}K\left(\frac{D}{U}\right)^{\frac{2\bar{\mu}}{\sigma^2}} N(z_1 - \sigma\sqrt{T}) \qquad (1)$$

where

$$z_1 = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{D^2 S_0}{U^2 K}\right) + \frac{\tilde{\mu}}{\sigma}\sqrt{T}.$$

If $K \le D$, the knock-in call option value is

$$DIC_u = S_0\left[\left(\frac{D}{U}\right)^{\frac{2\tilde{\mu}}{\sigma^2}} N(z_2) + \left(\frac{U}{S_0}\right)^{\frac{2\tilde{\mu}}{\sigma^2}} N(z_3) - \left(\frac{U}{S_0}\right)^{\frac{2\tilde{\mu}}{\sigma^2}} N(z_4)\right]$$
$$- e^{-rT}K\left[\left(\frac{D}{U}\right)^{\frac{2\bar{\mu}}{\sigma^2}} N(z_2 - \sigma\sqrt{T}) + \left(\frac{U}{S_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} N(z_3 + \sigma\sqrt{T}) - \left(\frac{U}{S_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} N(z_4 + \sigma\sqrt{T})\right]$$

where

$$z_2 = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{DS_0}{U^2}\right) + \frac{\tilde{\mu}}{\sigma}\sqrt{T}, \qquad z_3 = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{DS_0}{U^2}\right) - \frac{\tilde{\mu}}{\sigma}\sqrt{T}, \qquad z_4 = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{KS_0}{U^2}\right) - \frac{\tilde{\mu}}{\sigma}\sqrt{T}.$$

A down-and-out call option activated at the first passage time to the up-barrier is said to be a down-and out chained call (DOC_u). To value the DOC_u, the knock-in knock-out relation is used. Subtracting DIC_u from the up-and-in call price UICgivesDOC_u.

$DOC_u = UIC - DIC_u$.

An up-and-in doubly chained call (UIC_{ud}) is an up-and-in call option which is activated at time when the asset price crosses two different barrier levels (an up-barrier followed by a down-barrier). The formula of UIC_{ud} is also introduced in Jun and Ku[8].

III. Numerical Results

In this section we provide numerical results which compare the standard deviations of Monte Carlo simulation (MC) and Monte Carlo simulation withantithetic variate method(MC-Antithetic) for the chained optionDIC_u. Table 1 shows that these two simulation are similar to closed formula regardless of the conditions of up-barrier U and down-barrier D. The closed formula in Table 1 comes from equation (1). However, the standard deviation of Monte Carlo simulation withantithetic variate method is lower than Monte Carlo simulation for the chained option DIC_u when the up-barrier U and down-barrier Dhave various values. Option parameters are as in the following: The initial value $S_0 = 100$, the strike price $K = 100$, the volatility $\sigma = 0.3$, the time to maturity $T = 1$, the interest rate $r = 0.05$, monitoring frequency is 10,000, and the number of sample paths is 10,000.

As a result of the simulation, general Monte Carlo method needs to apply much more sample paths to maintain the standard deviation of Monte Carlo simulationwith antithetic variate method for chained option DIC_u. Furthermore, it is obviously time consuming. Therefore, Table 1 indicates that Monte Carlo simulation with antithetic variate method is much more efficient than general Monte Carlo method in chained option.

Table 1. Monte Carlo Simulation and Antithetic Variate Monte Carlo Simulation for

U	D	Closed Formula	MC	Std(MC)	MC-Antithetic	Std (MC-Antithetic)
105	80	0.9794	1.0221	5.4813	0.9708	3.6022
	85	2.3526	2.3430	8.4653	2.4137	5.8036
	90	4.8287	4.7474	12.5783	4.8276	8.2658
	95	8.7258	8.5793	17.2203	8.6424	10.5592
110	80	0.9421	0.9722	5.6365	0.9473	3.7303
	85	2.2968	2.2416	8.2829	2.2321	5.7124
	90	4.7635	4.6312	12.4881	4.7117	8.2619
	95	8.6677	8.7992	17.8518	8.6638	10.9205
115	80	0.8698	0.8837	5.1727	0.8763	3.6225
	85	2.1771	2.0674	8.1922	2.1532	5.8043
	90	4.6070	4.4814	12.2513	4.4641	8.0103
	95	8.5063	8.2824	17.5099	8.3528	10.8176
120	80	0.7723	0.7213	4.8281	0.7558	3.4708
	85	2.0040	1.9191	8.1424	1.9382	5.6266
	90	4.3521	4.2870	12.2954	4.1401	8.0165
	95	8.2109	7.9368	16.9515	8.0400	10.6507

Fig.1 represents the standard deviations of general Monte Carlo simulation and Monte Carlo simulation with variance reduction method. The simulation was executed under the assumption that $S_0 = 100, K = 100, U = 120, D = 80, \sigma = 0.3, T = 1$ and monitoring frequency is 10,000. Even though the number of sample paths increases from 10,000 to 100,000, we can see that the standard deviation of general Monte Carlo simulation is always bigger than that of Monte Carlo method with variance reduction method in case of chained option DIC_u.

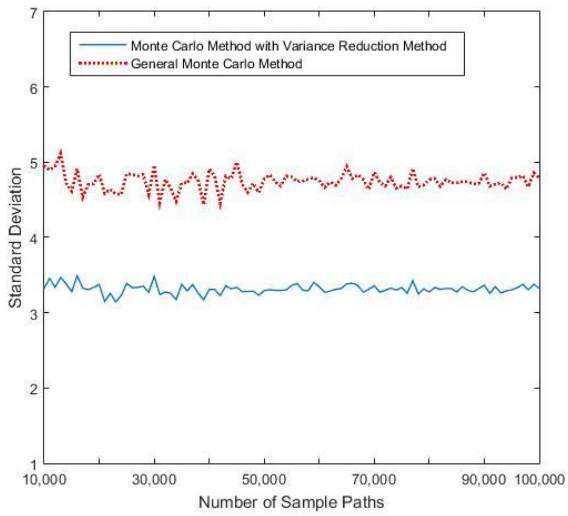

Figure1. Standard deviations of general Monte Carlo method and variance reduction Monte Carlo method for DIC_u

IV. Conclusion

In this paper, we simulated the General Monte Carlo method and Monte Carlo method with variance reduction method for chained option. We found that Monte Carlo method with variance reduction method is always more stable and efficient than General Monte Carlo method under the various variables which are up-barrier, down-barrier and sample paths for chained option.

References

[1] R.C. Merton, Theory of Rational Option Pricing, Bell Journal of Economics and Management Science, Vol. 4,1973, 141-183.
[2] E. Reiner, M. Rubinstein, Breaking down the barriers, Risk, Vol. 4(8),1991, 28-35.
[3] D. Rich, The mathematical foundations of barrier option pricing theory, Advances in Futures and Options Research, Vol. 7. 1997.
[4] N. Kunitomo, M. Ikeda, Pricing Options with Curved Boundaries,Mathematical Finance, Vol. 2,1992, 275-298.
[5] H. Geman, M. Yor, Pricing and Hedging Double-Barrier Options: A Probabilistic Approach, Math. Finance, Vol. 6,1996, 365-378.
[6] A. Pelsser, Pricing double barrier options using Laplace transforms,Finance and Stochastics 4, 2000, 95-104.
[7] D. Jun, H. Ku, Cross a Barrier to Reach Barrier Options, Journal of Mathematical Analysis and Applications, 389, 2012, 968-978.
[8] D. Jun, H. Ku, Pricing Chained Options with CurvedBarriers,Mathematical Finance, 23, 2013, 763-776.
[9] P.L'Ecuyer and C. Lemieux, Variance reduction via lattice rules, Management Science 46, 2000, 1214-1235.
[10] P. Glasserman, Monte Carlo Methods in Financial Engineering(Springer-Verla, 2004).
[11] J.C.Hull, Options, Futures, and Other Derivative Securities(Fourth ed.,Prentice Hall, Englewood Cliffs, NJ, 2005).