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A Minimum Spanning Tree Approach of Solving a Transportation Problem

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ABSTRACT: *This work centered on the transportation problem in the shipment of cable troughs for an underground cable installation from three supply ends to four locations at a construction site where they are needed; in which case, we sought to minimize the cost of shipment. The problem was modeled into a bipartite network representation and solved using the Kruskal method of minimum spanning tree; after which the solution was confirmed with TORA Optimization software version 2.00. The result showed that the cost obtained in shipping the cable troughs under the application of the method, which was AED 2,022,000 (in the United Arab Emirate Dollar), was more effective than that obtained from mere heuristics when compared.*
KEYWORDS: *Minimum spanning tree, Networking, routing, graph, spanning tree and optimal solution, Kruskal algorithm.*

I. INTRODUCTION

Transportation is one of the key or important unit in any organization. It may be difficult for this sector to be ignored in managerial decision making. This is because goods and other scarce resources will likely be distributed from one point to the other. Hence, a good or optimal planning is unavoidable or necessary so as to minimize the cost of transporting these items so as to maximize the profit for the organization. Researchers have done a lot in this area, yet much is still needed to be done. In this research, we have decided to utilize the concept of minimum spanning tree (MST) to provide the network of the cable trough of the research carried out by Basel et al (2015) which only provided an optimal solution using different soft wares without showing the solution in details.

II. LITERATURE REVIEW

Nicolic (2007) worked on total time-minimizing transportation problem using two variant functions that were time dependent. Both the efficiency of the transportation from time perspective and the total transportation time function were optimize via their routes. Kadhim et al (2015) proposed a new approach for solving transportation problem using modified kruskal algorithm. The approach was bias to graph theory while being supported by the kruskalalgorithm for finding minimum spanning tree (MST). This approach was able to provide a faster convergence criteria in meeting the minimum feasible solution.

Basel et al (2015) worked on a transportation plan where cable troughs were shipped from various source locations to their destination sites of construction. They were able to minimize cost but also observe that different model tools- Excel solver, Lingo/Lindo, MPL/Cplexetc, may result in different shipment allocation. However, all the models employed resulted in an optimal solution of AED80,000. Charles(2015) applied Prim`s algorithm for minimal spanning problem by designing a local area network in Chuka University, Kenya. He was able to minimize the total cost of the various University buildings interconnection which was represented by nodes with fibre-optic network..

Ibrahaim (2007) worked on graph algorithm and shortest path problems with application to the dual carriage ways in Sokoto metropolis using Djikstra`s algorithm. He was also able to obtain the minimum spanning tree (MST) for the problem. Arogundele et al (2011) employed Prim`s algorithm to model a local access network in Odeda local government in one of the states in Nigeria. A minimum spanning tree for the graph was generated for cost effective service within the local government. Donkoh et al (2011) applied Prim`s algorithm and the Steiner tree algorithms with factor rating method to obtain an optimal pipeline connection for the West African Gas pipeline project. Their result gave approximately 10.3% reduction of the original weight as collected from West African Gas pipeline (the organization responsible for the gas supply and pipeline).

III. APPROACH TO THE STUDY

(1) Represent the transportation as a network flow diagram
(2) Carry out iterations to obtain the minimum spanning tree of the network, using a special algorithm for MST
(3) Delineate all other possible spanning tree diagrams of the network, using simple heuristics
(4) Calculate and compare the objectives of all the spanning trees including that of the MST and justify the choice of the MST value as our optimal solution

(5) Develop a flow chart and a computer program that will solve the problem in real time and with optimal accuracy.

METHODOLOGY

This research deals with two different approaches to solving the transportation problem by the method of minimum spanning tree. But to proceed on this, we would first review the foundational concept of the transportation problem, formulating the model; consider various other methods of approaching it, and finally, the method chosen to obtain the optimal solution here.

Mathematical Formulation Of The Transportation Problem

The general problem can be approached in a manner described by the following:

Assuming we are to distribute commodity X from a number of factories where they are produced in a rural community setting (say m factories) to several warehouses in an urban area located at the center of their varying markets (say n warehouses). Suppose also we say that the total supply of goods to be shipped out from each factory is , and the total number of goods demanded by the various warehouse locations is b. Let the cost of distributing these goods between the various factories and the warehousesbe c. The following variables can be defined for the transportation problem:

a_i denotes the total of goods from factory i, where $i = 1,2,\dots,m$

b_j denotes the total demand on goods at warehouse j ; where $j = 1,2,\dots,n$

c_{ij} denotes the unit transportation cost from factory i to warehouse j

x_{ij} denotes the quantity of goods distributed from factory i to warehouse j .

If $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$

Then our objective function is,

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$$

Subject to the following constraints:

$\sum_{j=1}^{n} X_{ij} \le a_i$ (Supply constraint)

$\sum_{i=1}^{m} X_{ij} \ge b_j$ (Demand constraint)

$X_{ij} \ge 0$ (Non-negativity constraint)

$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (Balance constraint)

The general mathematical representation of the transportation problem is thus:

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$$

Subject to:

$\sum_{j=1}^{n} X_{ij} \le s_i \quad (\forall i\ ;\ i = 1,2,\dots,m)$ (1)

$\sum_{i=1}^{m} X_{ij} \ge d_j (\ \forall j\ ;\ j = 1,2,\dots,n\ )$ (2)

$X_{ij} \ge 0$ (3)

The summary of constraints (1) and (2), expressed as an inequality condition, is based on the assumption that there are several demand centers to be met by several supply centers in a typical transportation problem

IV. TRANSPORTATION TABLEAU

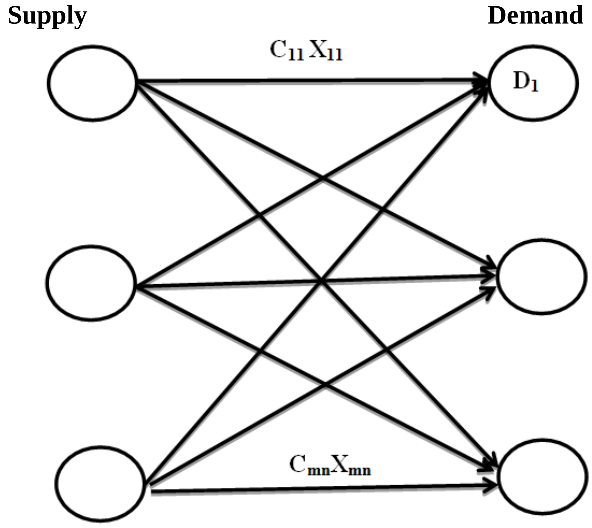
The transportation problem is here represented in a table called the transportation tableau.

Destinations (*j*)

SOURCES(*i*)	D_1	D_2	--------------	D_n	**SUPPLY(a_i)**
S_1	$C_{11}\ X_{11}$	$C_{12}\ X_{12}$	--------------	$C_{1n}\ X_{1n}$	a_1
S_2	$C_{21}\ X_{21}$	$C_{22}\ X_{22}$	--------------	$C_{2n}\ X_{2n}$	a_2
S_m	$C_{m1}\ X_{m1}$	$C_{m2}\ X_{m2}$	--------------	$C_{mn}X_{mn}$	a_m
Demand (b_j)	b_1	b_2	--------------	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b$

Table 1. Transportation Tableau

Network diagram of the transportation problem:
The transportation problem can also be presented as a network model with *m* supply nodes and *n* demand nodes.



Balanced And Unbalanced Transportation Problem

A typical case of a transportation problem in reality most often does not appear balanced–i.e. the total number of goods demanded does not always equal the total number of goods supplied ($\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$). Such form of transportation problem is called the unbalanced transportation problem, where equation (3) amongst the afore-listed constraint set is violated.

Such violation of the balance constraint presents itself in either of two different cases as we shall see in the following:

- $\sum_{i=1}^{m} a_I > \sum_{j=1}^{n} b$: this case is where we have the total supply exceeding the total demand. To correct this, dummy variables are added to the demand constraint to cover up the deficiency; and the difference $\sum_{i=1}^{m} a_I$ - $\sum_{j=1}^{n} b$ becomes the summed up demand for that dummy variable
- $\sum_{i=1}^{m} a_I < \sum_{j=1}^{n} b$: this second case of the unbalanced type of the transportation problem is the case where total supply is exceeded by total demand by some amount. To correct this, once again, we introduce dummy variables; and the difference $\sum_{i=1}^{m} a_i$ - $\sum_{j=1}^{n} b_j$becomes the capacity for the dummy supply center.

Obtaining The Optimal Solution

As we have earlier said, the method to be employed in this work is the method of minimum spanning tree. This method can best be approached algorithmically using either of two special algorithms for minimum spanning tree, called Kruskal algorithm and Prim algorithm.

In applying the methods, we work directly on the network model of our transportation problem, instead on the tableau as in the other popular methods.

Data Collection And Analysis

The aim of the transportation problem is to minimize cost of transportation of goods from source areas to destination locations as much as possible. The data used here for the analysis is secondary data.

V. DATA COLLECTION

The data used in this work was obtained from a work done by Basel et al (2015),Optimization Techniques in Civil Works and Electrical Contracting, as published under IEOM Society.

Problems illustrated here are of a real-life project in the field, covering the installation of underground cable circuits between two grid stations. Cable troughs were shipped from three different suppliers: Supplier 1, Supplier 2, and Supplier 3. They are collected at four different locations at the construction site viz: Sec1, Sec2, Sec3, and Sec4. Our aim here, using the Kruskal algorithm for minimum spanning tree, is to figure out which plan for assigning the shipments would minimize the total shipping cost, subject to the restrictions imposed by the fixed output from each supplier and the fixed allocation in each location at the site.

For this problem, the data that needed to be gathered included three categories: the output from each source (Supply), the allocation at each destination (Demand), the cost per unit shipped from each source to each destination (In AED). The procurement department at the company involved provided the data needed for the first and third categories above. Engineers supervising the work in each of the four locations at the site provided data of the second category. The cost from each supply end to each location depends on the distance, the nature of road, and the time of shipment.

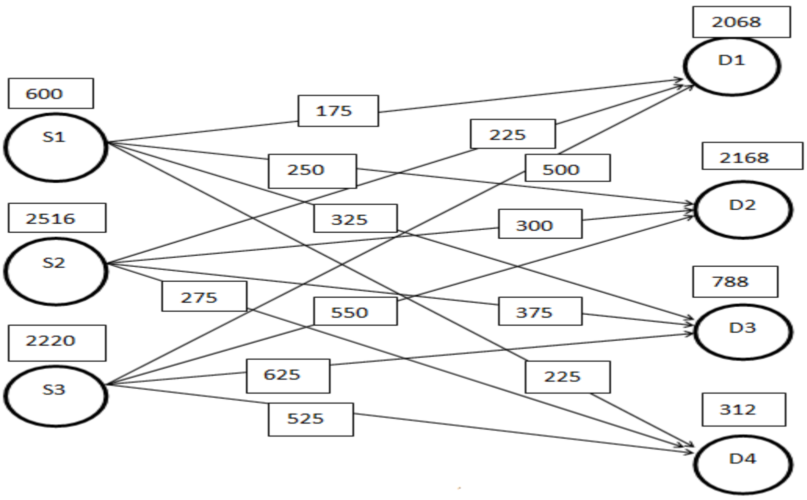
Since each trip can load four troughs, the shipping cost per trough can be found by dividing the shipping cost per trip by 4. This information of supply and demand (in units of cable troughs), along with the shipping cost per cable trough for each supplier-location combination is given in the Table below.

Table 2. Shipping data for the transportation problem

	LOCATION	**AT THE**	**SITES**		
SUPPLIERS	**SEC1**	**SEC2**	**SEC3**	**SEC4**	**CAPACITY**
AL MERAIKHY	175	250	325	225	600
UPC	225	300	375	275	2516
EPC	500	550	625	525	2220
ALLOCATION	2068	2168	788	312	**5336**

Table.3 The network representation of the problem is as shown below:

Demand/Supply points	AL MERAIKHY	UPC	EPC	SEC1	SEC2	SEC3	SEC4
Variable Names	S1	S2	S3	D1	D2	D3	D4



Applying Kruskal Algorithm On The Network

Fig.1

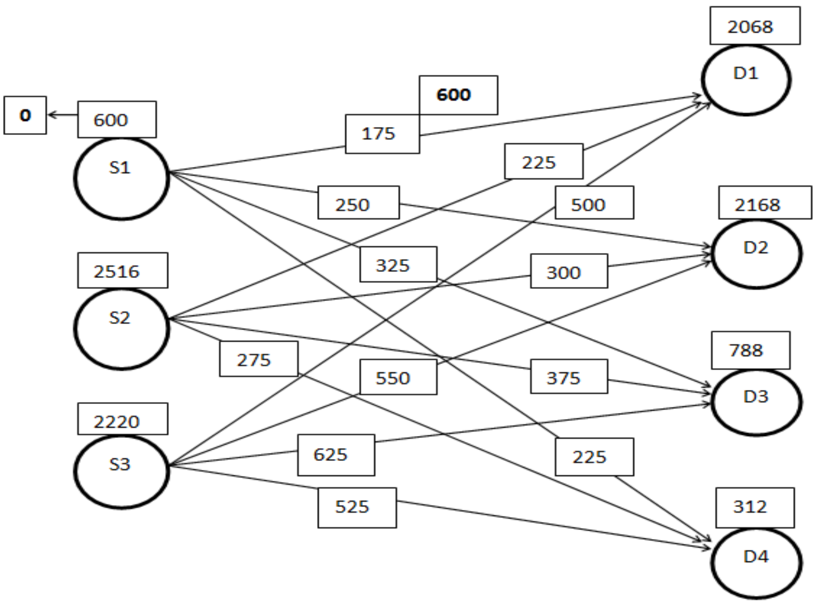


Fig.2

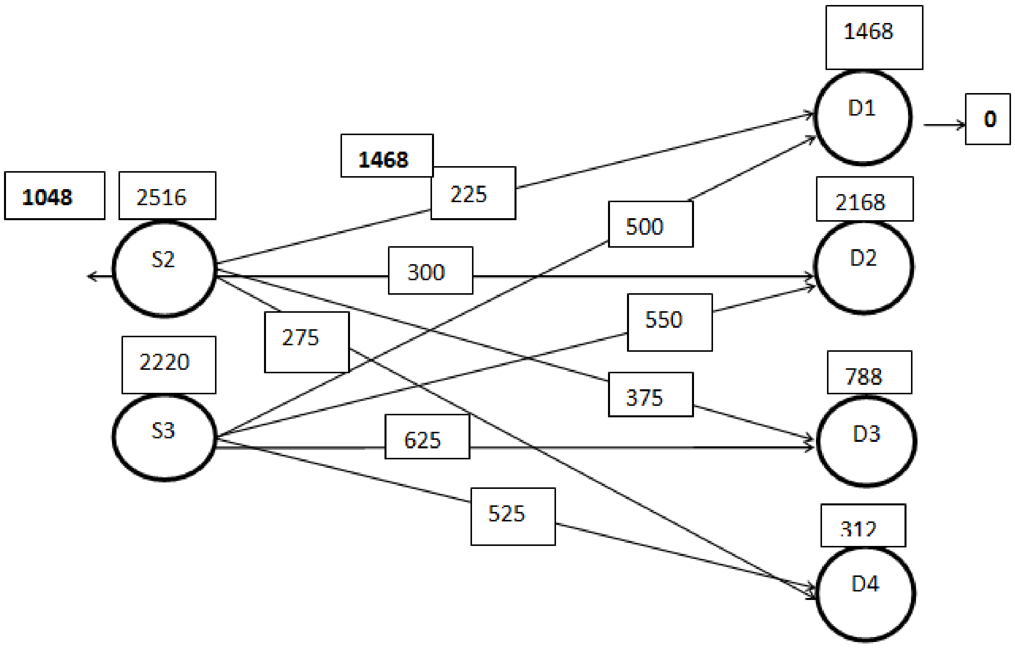


Fig.3

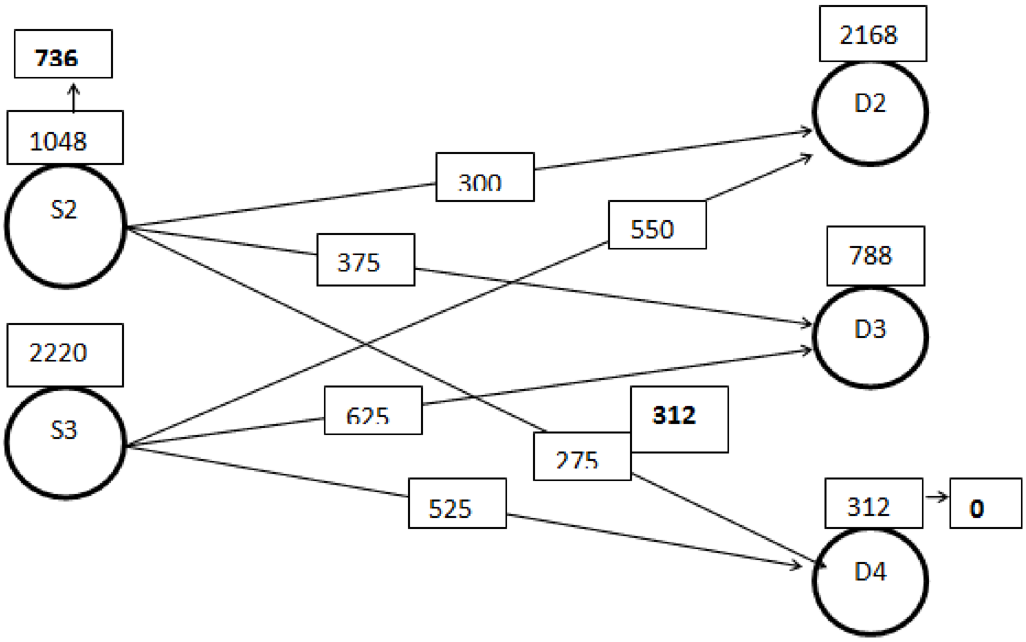


Fig.4

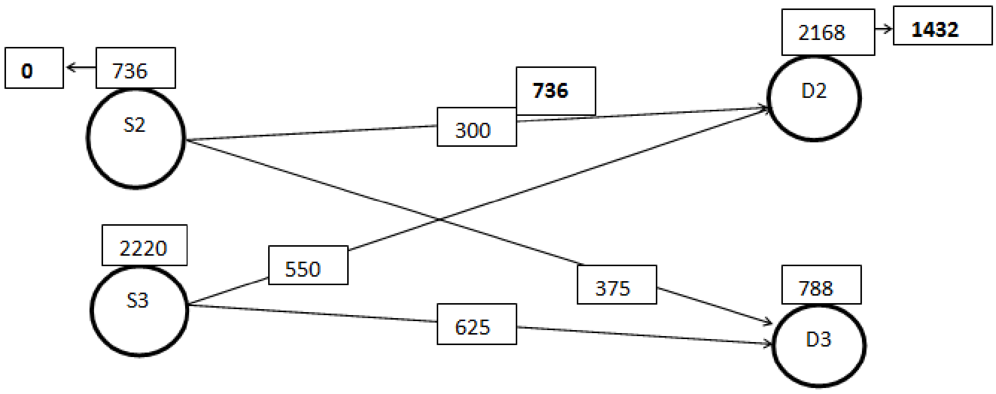


Fig. 5

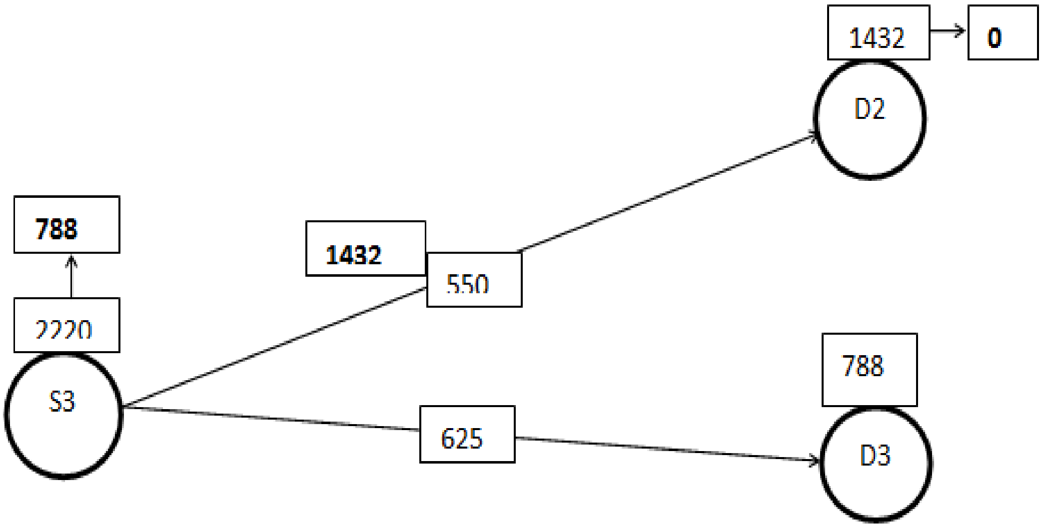
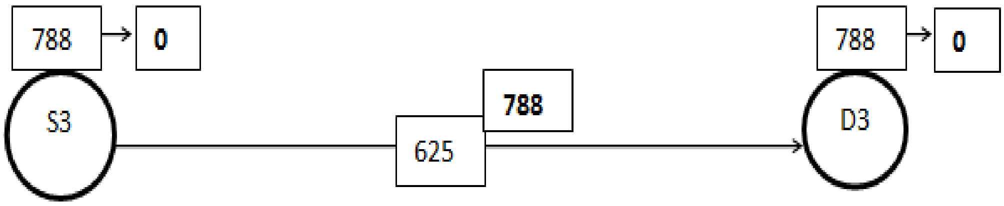


Fig. 6



The Minimum Spanning Tree:

Fig. 7

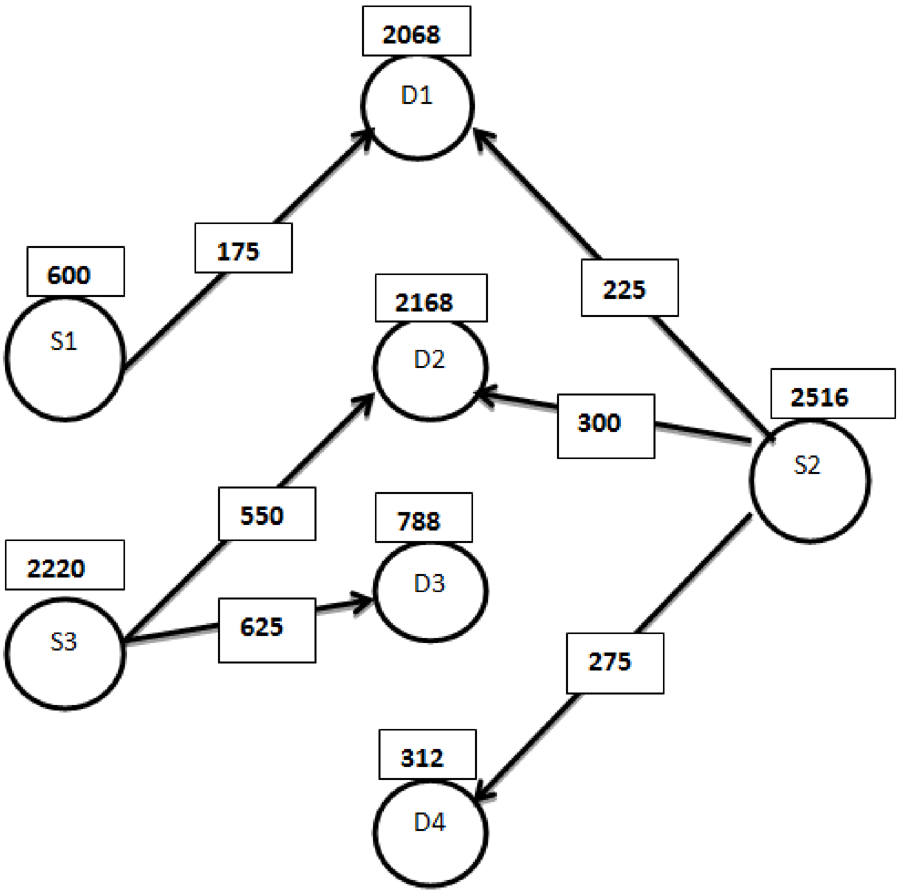


Table Representation of the Kruskal Algorithm

N	EDGE	COST	KRUSKAL STEP1 COST=175*600	KRUSKAL STEP2 COST=175*600+225*1468	KRUSKAL STEP3 COST=175*600+225*1468+275*312= 521100
1	S1D1	175	X	X	X
2	S1D4	225			
3	S2D1	225		X	X
4	S1D2	250			
5	S2D4	275			X
6	S2D2	300			
7	S1D3	325			
8	S2D3	375			
9	S3D1	500			
10	S3D4	525			
11	S3D2	550			
12	S3D3	625			
N	**EDGE**	**COST**	**KRUSKAL STEP4 COST=521100+300* 736=741900**	**KRUSKAL STEP5 COST=741900+550*1432= 1529500**	**KRUSKAL STEP6 COST=1529500+625*788=2022000**
1	S1D1	175	X	X	X
2	S1D4	225			
3	S2D1	225	X	X	X
4	S1D2	250			
5	S2D4	275	X	X	X
6	S2D2	300	X	X	X
7	S1D3	325			
8	S2D3	375			
9	S3D1	500			
10	S3D4	525			
11	S3D2	550		X	X
12	S3D3	625			X

MIN Z= $C_{12}X_{12}+C_{21}X_{21}+C_{24}X_{24}+C_{22}X_{22}+C_{32}X_{32}+C_{33}X_{33}$;

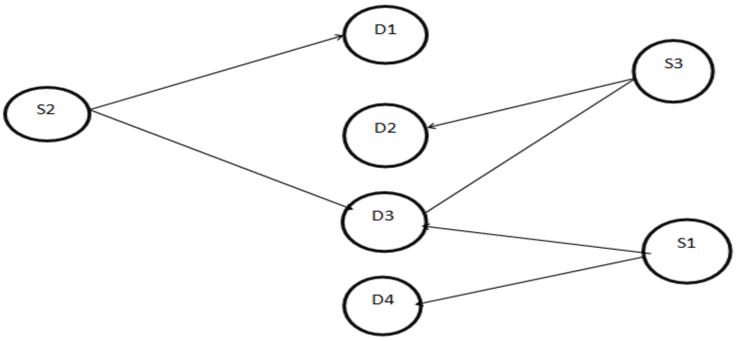
MIN Z= (175x600)+(225x1468)+(275x312)+(300x736)+(550x1432)+(625x788)

=2022000(AED)

A confirmation of the optimal solution as obtained in the TORA Optimization system Software version 2.00 is found in Appendix A.

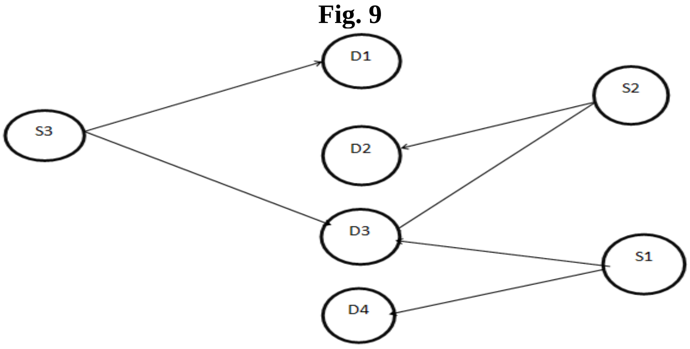
OTHER SPANNING TREES OF THE TRANSPORTATION CASE FROM MERE HEURISTICS:

Fig. 8



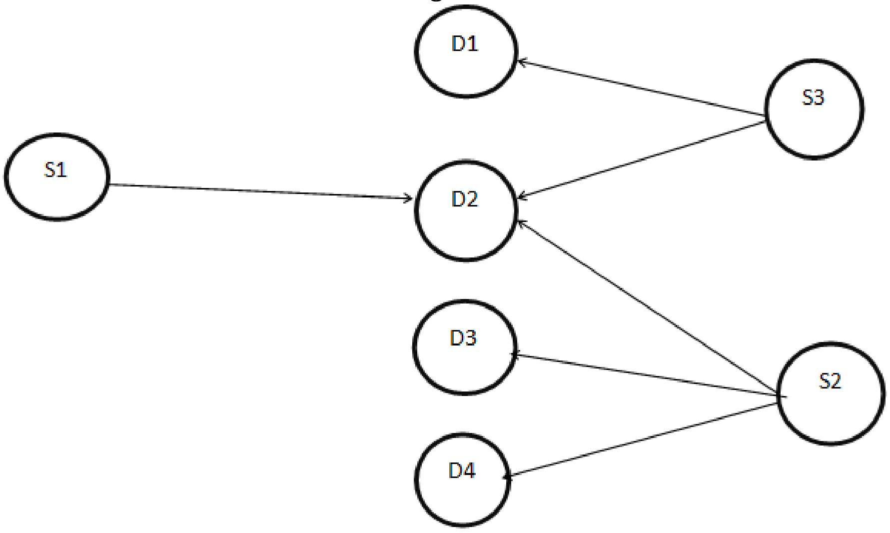
C_1= 2,710,900 (AED)

Fig. 9



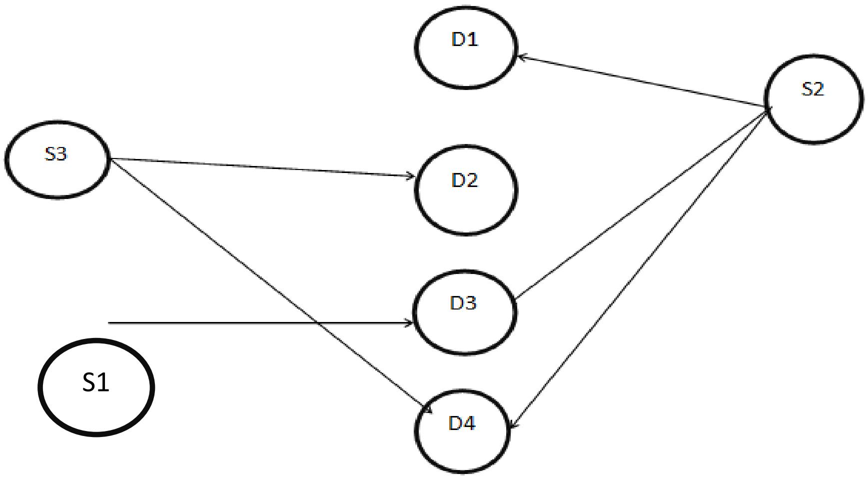
C_2= 2,737,600 (AED)

Fig. 10



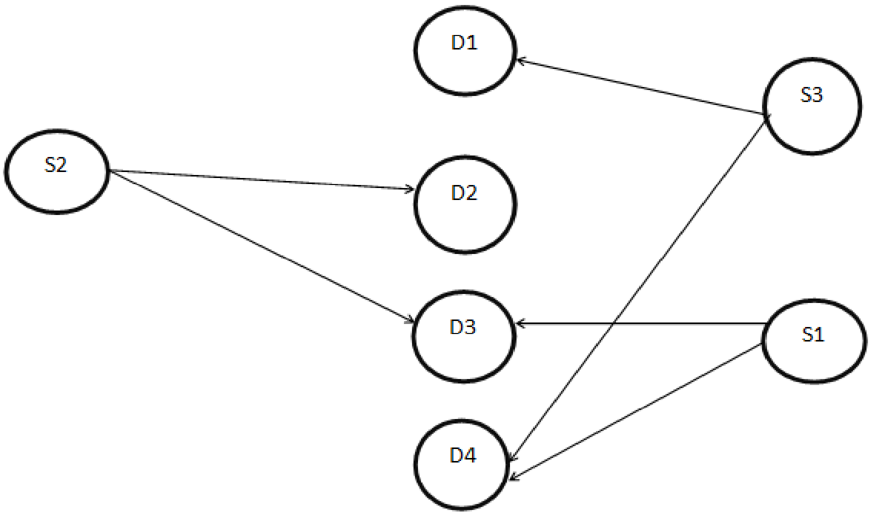
C_3= 3,408,100 (AED)

Fig. 11



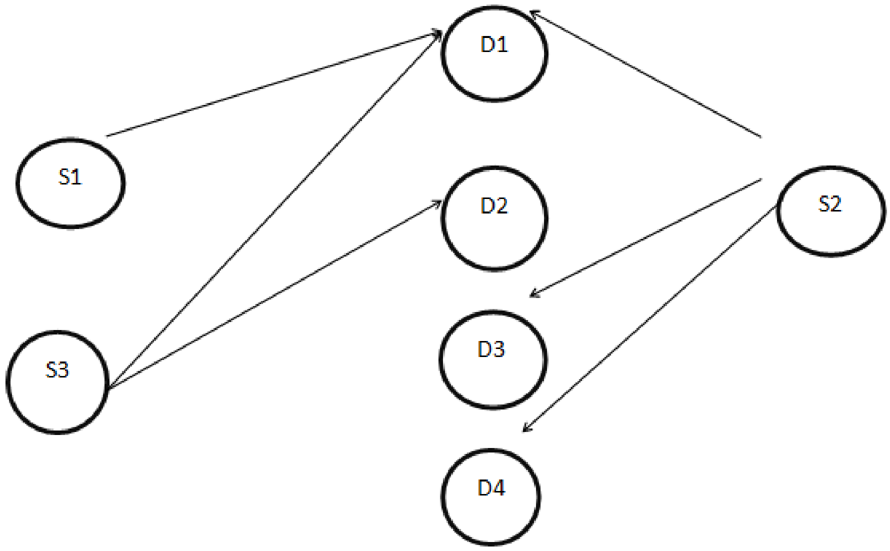
C_4= 2,397,800 (AED)

Fig.12



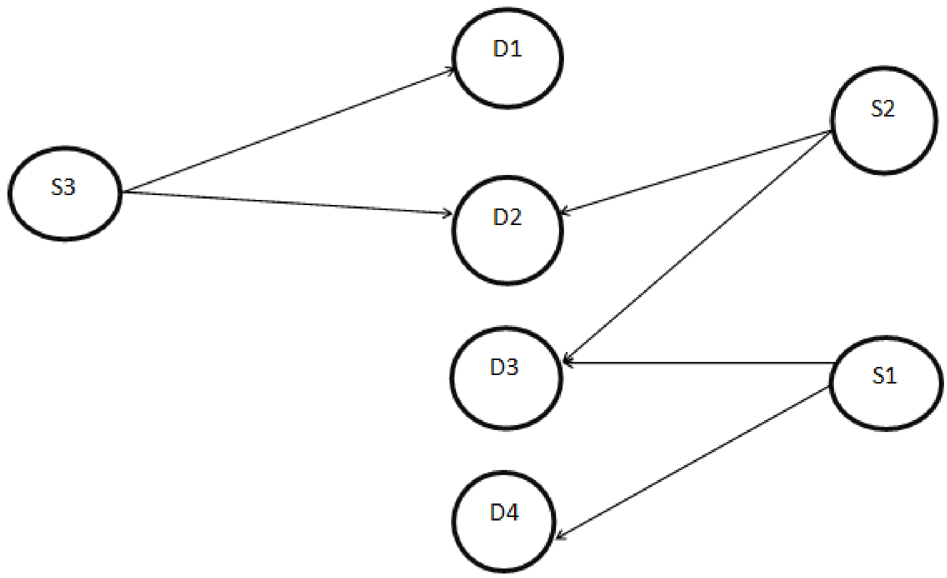
C_5= 2,560,100 (AED)

Fig. 13



C_6= 2,178,000 (AED)

Fig.14



C_7= 3,437,500 (AED)

Comparing All Spanning Trees (From Heuristics And Kruskal)
Assuming Min Z= 2,022,000= C_0, then we consider the following inequality distribution and justify our choice of choosing Kruskal spanning tree as our optimal solution:

$\mathbf{C_7>C_3>C_2>C_1>C_5>C_4>C_6>C_0}$

Therefore Fig.7 is our optimal solution to the Transportation problem.

VI. SUMMARY

Compared to the work from which our data was sourced, it was observed that the optimal solution obtained in the work was obtained directly from Excel Solver, without detail of exact routing; but in this work the optimal solution was arrived at graphically, indicating in details the routing pattern that led to it represented by trees and finally a spanning tree.

The logistics of the minimum spanning tree obtained here is described as thus: in order to achieve cost-effective distribution: Al Meraikhy (S1) must only deliver to Sec1 (D1); while UPC share's its delivery among Sec1, Sec2, and Sec4 (which are D1, D2, D4 respectively); then EPC, the third supply unit, is to share its delivery between Sec2 and Sec3 (D2 and D3). And the result gives an optimal solution of AED 2,022,000

VII. CONCLUSION AND RECOMMENDATION

The transportation problem remains a major problem in distribution firms, and as such should be effectively given managerial concern. Optimization of transportation cost is necessary as to meet profit goals in such firms; otherwise, the firm might run the risk of putting their organization gradually into bankruptcy. As we have observed in the work, mere heuristics cannot suffice to solve the transportation problem, but the application of effective techniques as we have presented here by the method of minimum spanning tree. With this method, we were able to optimize the cost of transporting cable troughs required for underground cable installation at a construction site, and the optimal solution gave AED 2,022,000; which, compared to mere heuristic approach, was the more cost-effective.

What this work has done in summary is to stimulate nonchalant firms which do not seek to optimize in order to achieve cost-effective services and grow the profit margin. It is here recommended that every growing enterprise which is into the distribution business should hence apply techniques like the Kruskal algorithm, so that they would be able to minimize the cost of distributing their goods or services.

REFERENCES

[1]. Aljanabi, K. B., &Jasim, A. N. (2015).An Approach for Solving Transportation Problem Using Modified Kruskal's Algorithm. *International Journal of Science and Research (IJSR)*, *4*(7), 2426-2429.

[2]. Arogundade, O. T., Sobowale, B., &Akinwale, A. T. (2011).Prim Algorithm Approach to Improving Local Access Network in Rural Areas. *International Journal of Computer Theory and Engineering*, *3*(3), 413.

[3]. Basel, A. A.,Amani, A., Khaled E., Mansoor A., &Rashed, A. (2015).Optimization Techniquesin Civil Works and Electrical Contracting.*Proceedings of the 2015 International Conference on Operations Excellence and Service Engineering Orlando, Florida, USA*,Sept. 10-11, 2015, 807-817.

[4]. Donkoh, E. K., Amponsah, S. K., &Darkwa, K. F. (2011).Optimal pipeline connection for the West African gas pipeline project. *Research Journal of Applied Sciences, Engineering and Technology*, *3*(2), 67-73.

[5]. Gitonga, C. K. (2015). Prims Algorithm and its Application in the Design of University LAN Networks. *International Journal*, *3*(10).

[6]. Ibrahim, A. A. (2007). Graph algorithms and shortest path problems: A case of Djikstra's algorithm and the dual carriage ways in Sokoto metropolis. *Trends Applied Sci. Res*, *2*, 348-353.

[7]. Nikolić, I. (2007). Total time minimizing transportation problem. *Yugoslav Journal of Operations Research*, *17*(1), 125-133.