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Finite Time Stability of Linear Control System with Constant Delay in the State

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ABSTRACT: *In this paper, the problem of finite time stability of a class of linear control systems with constant delay in the state is considered. Using Coppel's inequality and matrix measure sufficient delay dependent condition has been derived. The paper extends some basic results.*
KEYWORDS: *Linear control systems, finite time stability, delay dependent criteria, matrix measures*

I. INTRODUCTION

Time-delay is very often encountered in technical systems such as communications, robotic manipulator and pneumatic as well as process systems in chemical industries etc. ([1] and [2]). The presence of delays in the state or control is a source of instability for such system, hence stability analysis for such class of systems has been the interest of many researchers. Stability is an important property of any control system with or without delay, but the presence of delays in any system makes it relatively more difficult and much more complicated to analyze, see [3].

There are different kinds of stability problems that arise in the study of dynamical systems, such as Lyapunov stability, BIBO stability, finite time stability, practical stability etc. Lyapunov stability, asymptotic stability and other classical stability concepts deals with systems defined over an infinite time interval. These concepts requires the boundedness of the state variables, whereby the values of the bounds are not prescribed. In practice one is not only interested in stability of the system but also in the bounds of the system's trajectories. Therefore, classical stability concepts are inadequate for practical applications, because there are some cases where large values of the state are not acceptable. For example, for chemical processes in which it is of interest to maintain certain parameters such as temperature, humidity or pressure below a given threshold, or missile systems and space vehicles operating over a finite interval time, it's expected that their state variables be controlled within certain bounds. Therefore the study of stability problem for time delay systems is of theoretical and practical importance, see ([4], [5] and [6])

The concept of finite-time stability was first introduced in the sixties ([7] and [8]). A system is said to be finite-time stable if, given a bound on the initial conditions and a specified time interval, its state does not exceed a certain bound during this time interval. The concept of finite time stability studies the behavior of the system within a finite time interval, and requires the convergence of the solution in the specified finite time interval for some given initial conditions.

To verify the finite time stability of systems, several researchers have developed different techniques to investigate stability criteria. [9]obtained necessary and sufficient conditions for the finite time stability and finite time boundedness of linear systems subject to exogenous disturbance by means of operator theory.

In ([10] and [11])Lyapunov like method has been used to solve the problem of finite time and practical stability of a class of linear continuous time delay system.

([12] and [13]) obtained finite time stability results for some particular classes of time delay systems based on linear matrix inequality (LMI). Also ([5] and [14]) used LMI approach to obtained several stability conditions.

Further in [15] matrix measure approach is used to establish sufficient condition for the stability of linear dynamic systems over finite time interval. ([16] and [17]) used the Coppel's inequalities and matrix measures to investigate finite time stability of singular systems operating under perturbing forces.

Here we examine the problem of finite time stability for a class of linear control systems and presents sufficient condition that enables system trajectories to stay within the a priori given sets.

II. PRELIMINARY

Consider a linear control system with constant delay in the state:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau) + Bu(t) \quad (2.1)$$

$$x(t) = \varphi(t) \qquad t \in [-\tau, 0]$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control vector, A_0, A_1, and B are constant system matrices with appropriate dimensions and τ is constant time delay, ($\tau = const > 0$).

$\varphi \in C([-\tau,0],R^n)$ is an admissible initial state and $C([-\tau,0],R^n)$ is a Banach space of continuous function mapping the interval $[-\tau,0]$ into R^n which converges uniformly. The system behavior is defined over the time interval $J=[0,T]$, where T is a positive number.

For the time invariant sets $S_{(\ )}$, used as bounds of the system trajectories are assumed to be bounded, open and connected. Let S_β be the set of all allowable states of the system for all $t \in J$, S_α be the set of all initial states the system and S_γ be the set of all allowable control action such that they are a priori known and $S_\alpha \subseteq S_\beta$.

Before proceeding further, we will introduce the following definitions and theorems which will be used in the next section.

Matrix measures have been extensively studied in ([18] and [19] and it is used to estimate upper bounds of matrix exponential. Matrix exponentials $\exp(At)$ is also known as the state transition matrix in control theory, and its bounds are useful in the analysis and design of control systems. The following theorem relates an upper bound of a matrix exponentials to its matrix measures.

Theorem 2.1: ([18] and [20]) For any matrix $A \in R^{n\times n}$ the estimate

$$\left\|\exp\left(A(t)\right)\right\| \le \exp\left(\mu(A)(t)\right)$$

holds.

Theorem 2.2: [21] The matrix norm or Lozinskii logarithm norm of a $n \times n$ matrix A is

$$\mu(A)=\lim_{h\to 0}\frac{\left|I+hA\right|-I}{h}$$

where $\|(.)\|$ is any matrix norm compatible with some vector norm $|x|_{(.)}$. The matrix measure define in theorem 2.2 has three variants depending on the norm utilized in the definition ([18] and [22]).

$$\mu_1(A)=\max_k\left(\operatorname{Re}(a_{kk})+\sum_{\substack{i=1\\ i\neq k}}^{n}\left|a_{ik}\right|\right) \quad (2.2)$$

$$\mu_2(A)=\frac{1}{2}\max_i \lambda_i\left(A^T+A\right) \quad (2.3)$$

and

$$\mu_\infty(A)=\max_i\left(\operatorname{Re}(a_{ii})+\sum_{\substack{k=1\\ k\neq i}}^{n}\left|a_{ki}\right|\right) \quad (2.4)$$

Before stating our results, we introduce the concept of finite-time stability for time-delay system (2.1). This concept can be formalized through the following definition.

Definition 2.1: Time delayed control system is finite time stable with respect to $\{S_\alpha, S_\beta, T, \|(.)\|, \mu(A_0)\neq 0\}$, $\alpha<\beta$ if and only if :

$\varphi(t)\in S_\alpha,\ \forall t \in [-\tau,0]$ and $u(t)\in S_\gamma, \forall t \in T$

implies

$x(t:t_0,x_0)\in S_\beta, \forall t \in [0,T]$

See ([23] and [24])

It is assumed that the usual smoothness condition is satisfied by system (2.1) to ensure existence, uniqueness and continuity of solutions with respect to initial data.

III. MAIN RESULT

Definition: System defined by (2.1) satisfying the initial condition is finite time stable w.r.t. $\{T, \alpha, \beta, \gamma, \tau\}$, $\alpha < \beta$ if and only if:

$\|\varphi(t)\| < \alpha, \|u(t)\| < \gamma, \forall t \in T$ (3.1)

implies

$\|x(t)\| < \beta, \forall t \in T$ (3.2)

Theorem 3.1: System (2.1) with initial condition is said to be finite time stable with respect to $\{\alpha, \beta, \gamma, \tau, T, \mu(A_0) \neq 0\}$, if the following condition is satisfied:

$$e^{\mu(A_0)t} < \frac{\beta / \alpha}{1 + \mu^{-1}(A_0)\left(1 - e^{-\mu(A_0)\tau}\right)\left(\|A_1\| + \gamma\|B\|\right)}, \forall t \in [0, T] \quad (3.3)$$

where $\|(.)\|$ denotes the Euclidean norm.

Proof: The solution to equation (2.1) can be expressed in terms of the fundamental matrix as:

$$x(t) = \phi(t)\psi(0) + \int_{-\tau}^{0} \phi(t - s - \tau)A_1\psi(s)ds + \int_{-\tau}^{0} \phi(t - s)Bu(t - s)\psi(s)ds \quad (3.4)$$

Taking the norm of both sides of equation (3.4), we have

$$\|x(t)\| \le \|\phi(t)\|\|\psi(0)\| + \int_{-\tau}^{0} \|\phi(t - s - \tau)\|\|A_1\|\|\psi(s)\|ds + \int_{-\tau}^{0} \|\phi(t - s)\|\|B\|\|u(t - s)\|\|\psi(s)\|ds \quad (3.5)$$

Applying theorem 2.1 on equation (3.5) yields

$$\|x(t)\| \le e^{\mu(A_0)t}\|\psi(0)\| + \int_{-\tau}^{0} e^{\mu(A_0)(t-s-\tau)}\|A_1\|\|\psi(s)\|ds + \int_{-\tau}^{0} e^{\mu(A_0)(t-s)}\|B\|\|u(t - s)\|\|\psi(s)\|ds \quad (3.6)$$

Using condition (3.1) on equation (3.6) gives

$$\|x(t)\| \le \alpha e^{\mu(A_0)t}\left(1 + \|A_1\|\int_{-\tau}^{0} e^{-\mu(A_0)(s+\tau)}ds + \|B\|\int_{-\tau}^{0} e^{\mu(A_0)(t-s)}\|u(t - s)\|ds\right)$$

$$\le \alpha e^{\mu(A_0)t}\left[1 + \|A_1\|\left(\mu^{-1}(A_0)\right)\left(1 - e^{-\mu(A_0)\tau}\right) + \gamma\|B\|\left(\mu^{-1}(A_0)\right)\left(1 - e^{-\mu(A_0)\tau}\right)\right]$$

$$\le \alpha e^{\mu(A_0)t}\left[1 + \mu^{-1}(A_0)\left(1 - e^{-\mu(A_0)\tau}\right)\left(\|A_1\| + \gamma\|B\|\right)\right]$$

Applying the basic conditions of theorem 3.1, one obtains

$\|x(t)\| < \beta, \quad \forall t \in [0, T]$

Which completes the proof.

IV. AN ILLUSTRATIVE EXAMPLE

Given a time delay system

$\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + Bu(t)$

where

$$A_0 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

with initial state of the function

$$x(t) = \varphi(t) = 0 \qquad t \in [-\tau, 0]$$

It is obvious that by (2.3) $\mu(A_0) = 2$ and using $\|A\| = \sqrt{\lambda_{\max} A^T A}$, $\|A_1\| = 2$, $\|B\| = 1$

The system is finite time stable w.r.t $\{\alpha = 2, \beta = 50, \gamma = 10, \tau = 0.1, \mu(A_0) = 2\}$ where the maximal estimated time interval of finite time stability can be estimated using conditions of theorem (3.1) to obtain:

$$\left[1 + \frac{1}{2}(1 - 0.074)(2 + 10)\right]e^{2T_{est}^{\max}} < \frac{50}{2}$$

$$T_{est}^{\max} \approx 0.66\ s$$

V. CONCLUSION

Finite time stability is applicable where large values of the state are not acceptable. Here, sufficient conditions expressed as simple inequality, which guarantees finite time stability for a class of linear control systems with constant time delay in the state has been established.

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