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Analysis of Two-Echelon Inventory System with Two Suppliers

S. Mohamed Basheer [1] and K. Krishnan[2]
[1]*Research Scholar, PG& Research Department of mathematics,*
Cardamom Planters' Association College, Bodinayakanur, Tamil Nadu, India -625513
[2]*Assistant Professor, PG& Research Department of mathematics,*
Cardamom Planters' Association College, Bodinayakanur, Tamil Nadu, India -625513

ABSTRACT: *Inventories exist throughout the supply chain in various form for various reasons. Since carrying these inventories can cost anywhere from 20-40 % of their value a year, managing them in a scientific manner to maintain minimal levels makes economic sense. This paper presents a continuous review two echelon inventory system. The operating policy at the lower echelon is (s, S) that is whenever the inventory level traps to* ***s*** *on order for Q = (S-s) items is placed, the ordered items are received after a random time which is distributed as exponential. We assume that the demands accruing during the stock-out period are lost. The retailer replenishes their stock from the regular supplier which adopts (0,M) policy, $M = n_1Q$. When the regular supplier stock is empty the replacement of retailer stock made by the outside supplier who adopts (0, N) policy $N = n_2Q$. The joint probability disruption of the inventory levels of retailer, regular supplier and the outside supplier are obtained in the steady state case. Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of a numerical examples, which provide insight into the behaviour of the system are presented.*

KEY WORDS: *Continuous review inventory system, two-echelon, positive lead time.*

I. INTRODUCTION

Most manufacturing enterprises are organized in to network of manufacturing and distributed sites that procure Raw- material, process them into finished goods and distributed the finished goods in to customers. The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain)when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods.

The usual objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature. As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris (1915)**[7]**.Clark and Scarf (1960)**[4]** had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size, Recent developments in two-echelon models may be found in. One of the oldest papers in the field of continuous review multi-echelon inventory system is a basic and seminal paper written by Sherbrooke in 1968. Hadley, G and Whitin, T. M., (1963)**[6]**, Naddor .E (1966) **[12]** Inventory System, John Wiley and Sons, New York. Analysis of inventory systems, Prentice- Hall, Englewood Cliff, New Jersey. HP's(Hawlett Packard) Strategic Planning and Modeling(SPaM) group initiated this kind of research in 1977.Continuous review models of multi-echelon inventory system in 1980s concentrated more on repairable items in a Depot-Base system than as consumable items (see Graves, Moinzadeh and Lee). Kalpakam and Arivarignan(1988) introduced multiple reorder level policy with lost sales in inventory control system.

All these papers deal with repairable items with batch ordering. Jokar and Seifbarghy analyzed a two echelon inventory system with one warehouse and multiple retailers controlled by continuous review (R, Q) policy. A Complete review was provided by Benito M. Beamon (1998) **[2]**. Sven Axsater (1993) **[1]** proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

The supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf (1960).Continuous review perishable inventory models studied by Kalpakam. S and Arivarignan. G (1998)**[8]** and a continuous review perishable inventory system at Service Facilities was studied by Elango .C and Arivarignan .G,(2001)**[5].**

A continuous review (s, S) policy with positive lead times in two-echelon Supply Chain for both perishable and non perishable was considered by Krishnan. K and Elango. C. 2005. Krishnan .K And Elango .C. A continuous review (s, S) policy with positive lead times in two echelons Supply Chain was considered by Krishnan. K. (2007). Rameshpandy.M, et. al (2014)[**13**] consider a Two-Echelon Perishable Inventory System with direct and Retrial demands and Satheeshkumar.R (2014) [**14**] et. al consider a Partial Backlogging Inventory System in Two-echelon with Retrial and Direct Demands. The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. section 6 provides Numerical examples and sensitivity analysis.

II. MODEL

2.1 The Problem Description

The inventory control system in supply chain considered in this paper is defined as follows. A supply chain system consisting one Manufacturer(MF), two suppliers (regular and outside), single disruption centre(DC) and 'n' identical retailers dealing with a single finished product. These finished products moves from the manufacturer through the network consist of manufacture, supplier, DC, Retailers and the final customer.

A finished product is supplied from MF to supplier (regular and outside) which adopts (0, N) and (0,M) replenishment policy then the product is supplied to DC who adopts (s, S) policy. The demend at retailers node i follows an independent Poisson distribution with rate λ_i (i = 1, 2, ..., n). Scanner collect sales data (demand) at retailer nodes and Electronic Data Interchange (EDI) allows these data to be shared with DC. With the strong communication network and transport facility of unit of item is transferred to the corresponding retailer with negligible lead time. That is all the inventory transactions are managed by DC. The replacement of item in terms of product is made from regular supplier to DC is administrated with exponential distribution having parameter $\mu_{1(\geq 0)}$. The replenishment of items of pocket is made from manufacturer to outsources suppliers is instantaneous. Demands accruing during the stock out periods are assumed to be lost. The maximum inventory level at DC node S is fixed, and the recorder point is s and the ordering quantity is Q(=S-s) items. The maximum inventory at regular supplier in N(=nQ) and outsource supplier in M (=nQ)

2.2 Notations and variables

We use the following for the fourth coming analysis part of our theses

$(C)_{ij}$: The element of sub matrix at (i,j)th position of C

0 : Zero matrix

I : Identity matrix

e : A column vector of 1's of appropriate dimension

λ_i : Mean arrival rate at the retailer *i*

λ : Demand rate at DC ($\lambda = \lambda_1 + \lambda_2 + ... + \lambda_n$)

μ_1 : Mean replacement rate at retailer

μ_2 : Mean replacement rate at regular supplier

S : Maximum inventory level at retailer

s : Minimum inventory level at retailer

N : Maximum inventory level at regular supplier

M : Maximum inventory level at outsources supplier

H_r : Holding cost at regular

H_{rd} : Holding cost at regular supplier

H_{od} : Holding cost at outsources supplier

O_r : Ordering cost at retailer

O_{rd} : Ordering cost at regular supplier

O_{od} : Ordering cost at outsources supplier

P_r : penalty cost at retailer

P_{rd} : penalty cost at regular supplier

I_r : Average inventory level at retailer

I_{rd} : Average inventory level at regular supplier

I_{od} : Average inventory level at outsource supplier

R_R : Mean reorder rate at retailer

R_{Rd} : Mean reorder rate at regular supplier
R_{od} : Mean reorder rate at outsource supplier
S_R : Shortage rate at retailer
S_{Rd} : Shortage rate at regular supplier
$\sum_{i=Q}^{nQ}$: Q + 2Q + 3Q + ... + nQ

III. ANALYSIS

Let I_1 (t), I_2 (t) and I_D (t) denote the on hand inventory levels of outside suppliers, regular suppliers and Distributor respectively at time t^+.

We define I (t) = { (I_1 (t), I_2 (t), I_D (t),) : t ≥ 0 } as a Markov process with state space E = { (i, j, k) | i = Q,.....nQ, j = Q , ...nQ , k= 0, ...S}. Since E is finite and all its states are aperiodic , recurrent non- null and also irreducible . That is all the states are ergodic. Hence the limiting distribution exists and is independent of the initial state

The infinitesimal generator matrix of this process C = (a (i, j, k, : l, m, n))$_{(i,j,k)(l,m,n)\in E}$ can be obtained from the following arguments.

- The arrival of a demand at distributor make a state transition in the Markov process from (i, j, k) to (i, j, k-1) with the intensity of transition $\lambda > 0$.
- The replacement of inventory at distributor from regular supplier makes a state transition from (i, j, k) to (i, j-Q, k+Q) with intensity of transition $\mu_2 > 0$.
- The replacement of inventory at distributor from outside supplier makes a state transition from (i, j, k) to (i-Q, j, k+Q) with intensity of transition $\mu_1 > 0$.

The infinitesimal generator C is given by

$$C = \begin{bmatrix} A & B & 0 & \dots & 0 & 0 \\ 0 & A & B & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & A & B \\ B & 0 & 0 & \dots & 0 & A \end{bmatrix}$$

Hence entries of C is given by

$$[C]_{pq} = \begin{cases} A & p = q; \quad q = nQ, \quad (n-1)Q, \dots Q \\ B & p = q + Q; \quad q = (n-1)Q, \dots .Q \\ B & p = q - (n-1)Q \quad q = nQ \\ 0 & otherwise \end{cases}$$

The sub matrices are given by

$$[A]_{pq} = \begin{cases} A_{11} & p = q; \quad q = nQ, \quad (n-1)Q, \dots Q \\ A_{12} & p = q + Q; \quad q = (n-1)Q, \dots .Q \\ A_{13} & p = q - (n-1)Q \quad q = nQ \\ A_{14} & p = q \quad q = 0 \\ 0 & otherwise \end{cases}$$

$$[B]_{pq} = \begin{cases} A_{12} & p = q; q = 0 \\ 0 & otherwise \end{cases}$$

The sub matrices of A and B are

$$[A]_{11} = \begin{cases} \lambda & p = q; \mathrm{q} = \mathrm{S}, \ldots\ldots \mathrm{s}+1 \\ -(\lambda + \mu_1) & p = q; q = s, \ldots\ldots\ldots\ldots 1 \\ -\mu_1 & p = q; q = 0 \\ \lambda & p = q+1; q = S-1, \ldots.1, 0 \\ 0 & otherwise \end{cases}$$

$$[A]_{12} = \begin{cases} \mu_1 & p = q - Q; \mathrm{q} = \mathrm{S}, \mathrm{S}-1 \ldots.. \mathrm{Q} \\ 0 & otherwise \end{cases}$$

$$[A]_{13} = \begin{cases} \mu_2 & p = q; \mathrm{q} = \mathrm{S}, \ldots..0 \\ 0 & otherwise \end{cases}$$

$$[A]_{14} = \begin{cases} -(\lambda + \mu_2) & p = q; \mathrm{q} = \mathrm{S}, \ldots\ldots \mathrm{s}+1 \\ -(\lambda + \mu_1 + \mu_2) & p = q; q = s, \ldots\ldots\ldots\ldots 1 \\ -(\mu_1 + \mu_2) & p = q; q = 0 \\ \lambda & p = q+1; q = S-1, \ldots.1, 0 \\ 0 & otherwise \end{cases}$$

3.1 Steady State Analysis

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process $\{ \mathrm{I}(t) : t \geq 0 \}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be $\Pi^{i}_{j,k} = \lim_{t \to \infty} \Pr\{(I_1(t), I_2(t), I_3(t) = (i, j, k)\}$ where $\prod^{i}_{j,k}$ is the steady state probability that the system be in state *(i, j, k)*. Let $\Pi = \left\{ \prod^{nQ}_{j,k}, \prod^{n-1}_{j,k}, \ldots.. \prod^{nQ}_{j,k} \right\}$ denote the steady state probability distribution. For each $((i, j, k)$, $\prod^{i}_{j,k}$ can be obtained by solving the matrix equation Π C = 0 together with normalizing condition $\sum_{(i,j,k) \in E} \prod^{i}_{j,k} = 1$

IV. OPERATING CHARACTERISTIC

In this section we derive some important system performance measure.

4.1 Average inventory Level

The event $I_R, \mathrm{I}_{Rd}, \mathrm{I}_{Od}$ denote the average inventory level at Distributor, regular supplier, and outsource supplier respectively,

(i) $$I_R = \sum_{i=Q}^{nQ^*} \sum_{j=0}^{nQ^*} \sum_{k=0}^{S} k \prod^{i}_{j,k}$$

(ii) $$I_{Rd} = \sum_{i=Q}^{nQ^*} \sum_{k=0}^{S} \sum_{j=0}^{nQ^*} j \prod^{i}_{j,k}$$

(iii) $I_{Od} = \sum_{i=Q}^{nQ^*} \sum_{k=0}^{S} \sum_{j=0}^{nQ^*} j \prod_{j,k}^{i}$

4.2 Mean Reorder Rate

Let R_R, R_{Rd}, R_{Od} be the mean reorder rate at retailer, regular supplier, outsource supplier respectively,

(i) $R_R = \lambda \sum_{i=Q}^{nQ^*} \sum_{j=Q}^{nQ^*} \prod_{j,s+1}^{i}$

(ii) $R_{Rd} = \mu_1 \sum_{i=Q}^{nQ^*} \sum_{k=0}^{S} \prod_{Q,k}^{i}$

(iii) $R_{Od} = \mu_1 \sum_{j=Q}^{nQ^*} \sum_{k=0}^{S} \prod_{j,k}^{Q}$

4.3 Shortage rate

Let $S_{R,} S_{Rd}$ be the shortage rate at retailer and regular supplier

(i) $S_R = \lambda \sum_{i=Q}^{nQ^*} \sum_{j=Q}^{nQ^*} \prod_{j,0}^{i}$

(ii) $S_{Rd} = \mu_1 \sum_{i=Q}^{nQ^*} \sum_{j=Q}^{nQ^*} \prod_{0,k}^{i}$

V. COST ANALYSIS

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate C(S, Q) is given by

$$c(s,Q) = (H_r * I_r) + (H_{rd} * I_{rd}) + (H_{od} * I_{od}) + (O_r * R_r) + (O_{rd} * R_{rd}) + (O_{od} * R_{od}) + (P_r * S_r) + (P_{rd} * S_{rd})$$

Although we have a not proved analytically the convexity of the cost function C(S,Q) our experience with considerable number of numerical examples indicate that C(s, Q) for fixed Q appears to be convex s. In some cases it turned out to be increasing function of s. For large number case of C(s,Q) revealed a locally convex structure. Hence we adopted the numerical search procedure to determine the optimal value of s.

VI. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

6.1 Numerical Example

In this section we discuss the problem of minimizing the structure. We assume Hr ≤ Hrd ≤ Hod the holding cost at distribution node is less than that of regular distributor node and an outside distributor node. Holding cost at the regular distributor node is less than outsource distributor node as the rental charge may be high at outsource distributor. Also Or ≤ Ord ≤ Ood the ordering cost at retailer node is less than that of regular distributor node and an outsource distributor node. Ordering cost at the regular distributor is less than outsource distributor node. Pr ≤ Prd the penalty cost at the retailer node is less than that of regular distributor.

The results we obtained in the steady state case may be illustrated through the following numerical example,

S = 16, M = 80, N =60, λ = 4, $\mu_1 = 3, \mu_2 = 2$ $H_r = 1.1,\ H_{rd} = 1.2,\ H_{od} = 1.3$

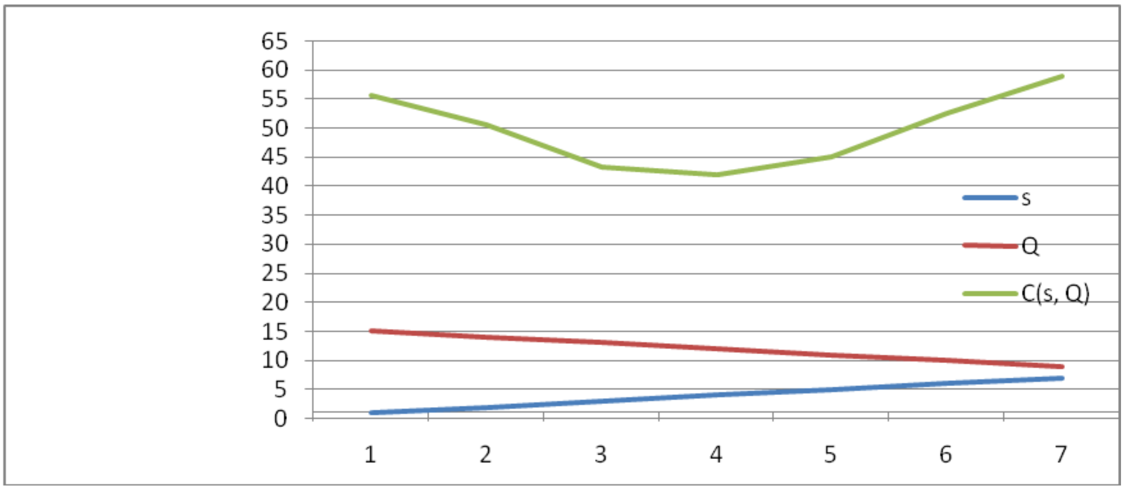
$O_r = 2.1,\ O_{rd} = 2.2,\ O_{od} = 2.3\ \ P_r = 3.1,\ P_{rd} = 3.2,$

The cost for different reorder level are given by

s	1	2	3	4*	5	6	7
Q	15	14	13	12	11	10	9
C(*s*, Q)	55.5433	50.3935	43.1193	41.8575*	44.8999	52.5676	58.8888

Table: 1 Total expected cost rate as a function s and Q

For the inventory capacity S, the optimal reorder level s* and optimal cost C(s, Q) are indicated by the symbol *. The Convexity of the cost function is given in the graph.



5.2 Sensitivity Analysis

Below tables are represented a numerical study to exhibit the sensitivity of the system of the system on the effect of varying demands rate

λ & $\mu_{1,}\mu_1$ & μ_2, H_r & $H_{rd;}$ P_r & $P_{rd;}$ P_r & $H_{r;}$ with fixed reorder at s = 5.

Table: 2 Effect on Replenishment rate& Demand rates(λ & $\mu_{1,}$)

$\mu_1 \backslash \lambda$	**1**	**2**	**3**	**4**	**5**
1	1.7561	4.1303	7.1513	10.2350	13.3300
2	2.5075	3.1511	5.1768	7.9816	10.9915
3	3.2679	3.9399	4.7246	6.5472	9.1500
4	3.9023	4.9912	5.5337	6.4597	8.2151
5	4.4775	5.9365	6.7085	7.3080	8.3419

Table: 3 Effect on Replenishment rates $\mu_1 \backslash \mu_{2,}$

$\mu_1 \backslash \mu_{2,}$	**1**	**3**	**5**	**7**	**9**
1	10.3286	7.3042	8.8128	12.0474	15.1632
3	10.2024	6.2966	6.7129	8.9687	11.1546
5	10.1761	6.0605	6.1958	8.2014	10.1526
7	10.1647	5.9555	5.9620	7.8535	9.6979
9	101585	5.8961	5.8288	7.6547	9.4377

Table 4: Effect on Holding cost ($H_r \backslash H_{rd,}$)

$H_r \backslash H_{rd,}$	**1.1**	**2.1**	**3.1**	**4.1**	**5.1**
1.1	6.5519	6.6374	6.7162	6.7951	6.8740
2.1	6.7159	6.7948	6.8734	6.9526	7.0314
3.1	6.8734	6.9523	7.0311	7.1100	7.1889
4.1	7.0308	7.1094	7.1886	7.2674	7.3463
5.1	7.1883	7.2671	7.3460	7.4249	7.5038

Table 5: Effect on Ordering Cost ($O_r \backslash O_{rd,}$)

$O_r \backslash O_{rd,}$	**2.1**	**2.2**	**2.3**	**2.4**	**2.5**
2.1	6.5519	6.5742	6.5929	6.6135	6.6340
2.2	6.5742	6.5948	6.6153	6.2456	6.6564
2.3	6.5966	6.6172	6.6377	6.6582	6.6788
2.4	6.6190	6.6172	6.6601	6.6806	6.7012
2.5	6.6414	6.6619	6.6825	6.7030	6.7235

Table 6: Effect on Penalty Cost (P_r \ $P_{rd,}$)

P_r \ $P_{rd,}$	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**
3.1	6.5424	6.6578	6.7732	6.8866	7.0040
3.2	6.5742	6.6896	6.8051	6.9205	7.0359
3.3	6.6061	6.7215	6.8369	6.9523	7.0677
3.4	6.6380	6.7534	6.8688	6.9841	7.0996
3.5	6.6699	6.7853	6.9007	7.61	7.1315

Table 7: effect on Holding cost & ordering cost(H_{od} \ $O_{od,}$)

H_{od} \ $O_{od,}$	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**
1.1	6.4920	6.5087	6.5244	6.5402	6.5559
1.2	6.5178	6.5338	6.5493	6.5651	6.5808
1.3	6.5428	6.5585	6.5742	6.5900	6.6057
1.4	6.5677	6.5834	6.5992	6.6149	6.6306
1.5	6.5926	6.6083	6.6241	6.6398	6.6555

Table 8: effect on Holding cost & penalty cost (H_r \ $P_{r,}$)

H_r \ $P_{r,}$	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**
3.1	6.5742	6.5821	6.5900	6.5979	6.6058
3.2	6.6892	6.6896	6.7054	6.7133	6.7212
3.3	6.8051	6.8129	6.8208	6.8287	6.8366
3.4	6.9205	6.9283	6.9362	6.9441	6.9520
3.5	7.0359	7.0438	7.0516	7.0595	7.0674

It is observed that from the table , the total expect cost C (s, Q) is increases with the different demand rates. Hence the demand rate is very important parameter of this system.

VII. CONCLUSION

This paper deals with an Inventory problem with two supplier, namely a regular supplier and outside supplier. The demand at retailer i follows independent Poisson with rate λ_i. The sum of 'n' independent demands with rate λ_i (i = 1,2,...n) is again a Poisson distribution with rate $\lambda = [\lambda 1\ \lambda 2 + \ldots + \lambda n]$. The structure of the chain allows vertical movement of goods from to regular supplier to Retailers. If there is no stock in regular supplier, then the DC will get products from outside supplier. The model is analyzed within the framework of Markov processes. Joint probability distribution of inventory levels at DC, Regular and Outside suppliers in the steady state are computed. Various system performance measures are derived and the long-run expected cost rate is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate. It would be interesting to analyze the problem discussed in this paper by relaxing the assumption of exponentially distributed lead-times to a class of arbitrarily distributed lead-times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special eases.

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