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# On Quasi Conformally Flat Sp-Sasakian Manifolds

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***Abstract:*** *In a SP-Sasakian Manifold, quasi conformally at reduces to a manifold of constant curvature and conversly. Further a quasi conformally at SP-Sasakian manifold with R(X,Y).S= 0is an Einstein manifold. Further , the Ricci tensor S has two distinct non-zero eigen values.*

## I. INTRODUCTION

Several Mathematicians [1], [2], [3] has studied SP-Sasakian manifold. We have studied such a manifold where Quasi Conformally Curvature tensor vanishes. It has been shown that the manifold reduces to a space of constant curvature; provided a+b(n-2)$\neq$0. It has also been shown that a SP-Sasakian manifold with constant curvature is quasi conformally flat. Finally a SP-Sasakian manifold with R(X,Y).S=0 has been studied. It has been shown that such a manifold reduces to an Einstein manifold. Further, the Ricci tensor S has two distinct non-zero eigen values.

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**Key words:**SP-Sasakian manifold,Curvature,Eigen values

**Preliminaries**

Let M be an almost paracontact metric manifold of dimension n with an almost paracontact metric structure ($\phi$,$\xi$,$\eta$,g) where $\phi$ is a 1-1 tensor ,$\xi$ is a contravariant vector field,$\eta$ is a 1-form and g is an associated Riemannian metric on M such that

1.1) $\phi^2 = I - \eta \otimes \xi, \eta(\xi) = 1, \phi\xi = 0, \eta o \phi = 0,$

1.2) $g(X,Y) = g(\phi X, \phi Y) + \eta(X)\ \eta(Y),\ \eta(X) = g(X,\xi)\ \forall$ X,Y

Such M is called a P-Sasakian manifold [1],[2],[3] provided

$(\nabla_X^{\phi})Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi$

From 1.3) it follows that

1.4) $\nabla_X^{\xi} = \phi X,\ (\nabla_X^{\eta})Y = g(\phi X, Y)$

It is known that

1.5) R(X,Y)$\xi$ =$\eta$(X)Y $-$ $\eta$(Y)X

1.6) $R(\xi, X)Y = \eta(Y)X - g(X,Y)\xi$

1.7) $S(X,\xi) = -(n-1)\eta$(X)

1.8) $Q\xi = -(n-1)\xi$

A P-Sasakian manifold is said to be a SP-Sasakian manifold([1],[2],[3]) if it satisfies

1.9) $g(\phi X, Y) = -g(X,Y) + \eta(X)\ \eta(Y)$

The quasi conformal curvature tensor [4] in a Riemannian manifold is a (1-3) tensor , denoted by $\tilde{C}$ and is defined as follows:

1.10) $\tilde{C}$(X,Y)Z= a R(X,Y)Z + b$\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\}$

$$-\frac{r}{n}(\frac{a}{n-1} + 2b)\{g(Y,Z)X - g(X,Z)Y\}$$

Here we have assumed

1.11) $\tilde{C}$(X,Y)Z =0

together with

1.12) $R(X,Y).S = 0$

where R(X,Y) denotes the derivation of the tensor algebra at each point of the manifold for tangent vectors X,Y and S is the non-zero Ricci tensor such that

1.13) $g(QX,Y) = S(X,Y)$

where Q is the symmetric endomorphism of the tangent space at M.

## II. QUASI CONFORMALLY AT SP-SASAKIAN MANIFOLD

From 1.10) and 1.11) we see that

2.1) $R(X,Y)Z = -\frac{b}{a}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\}$

$$+\frac{r}{an}(\frac{a}{n-1} + 2b)\{g(Y,Z)X - g(X,Z)Y\}$$

Using 1.13) we find

2.2) $g(R(X,Y)Z,U) = -\frac{b}{a}\{S(Y,Z)g(X,U) - S(X,Z)g(Y,U) + g(Y,Z)g(QX,U) - g(X,Z)g(QY,U)$

$$+\frac{r}{an}(\frac{a}{n-1} + 2b)\{g(Y,Z)g(X,U) - g(X,Z)g(Y,U)\}$$

Putting X=U=$\xi$ and using 1.1),1.6),1.7), we get from above

2.3) $S(Y,Z)=\{\frac{r}{bn}(\frac{a}{n-1} + 2b) + (n-1) + \frac{a}{b}\}g(Y,Z) - \{\frac{a}{b} + 2(n-1) + \frac{r}{bn}(\frac{a}{n-1} + 2b)\}\eta(Y)\eta(Z)$

2.4) $QY = \{\frac{r}{bn}(\frac{a}{n-1} + 2b) + (n-1) + \frac{a}{b}\}Y - \{\frac{a}{b} + 2(n-1) + \frac{r}{bn}(\frac{a}{n-1} + 2b)\}\eta(Y)\xi$

Contracting 2.3) we get

$-\frac{r}{n}\{a + b(n-2)\} = (n-1) + b(n-2)\}$

2.5) $r = -n(n-1)$, provided a+b(n-2)$\neq 0$

Putting 2.5) in 2.4) we get

2.6) $QY = -(n-1)Y$

Thus

2.7) $S(Y,Z) = -(n-1)g(Y,Z)$

Consequently 2.1) reduces to

$$R(X,Y)Z = -\frac{b}{a}\{-(n-1)g(Y,Z)X + (n-1)g(X,Z)Y - (n-1)g(Y,Z)X + (n-1)g(X,Z)Y\}$$

$$-\frac{n-1}{a}(\frac{a}{n-1} + 2b)\{g(Y,Z)X - g(X,Z)Y\}$$

2.8) $R(X,Y)Z = -\{g(Y,Z)X - g(X,Z)Y\}$

**Theorem 1:** A quasi conformally flat SP-Sasakian manifold is of constant curvature of value -1, provided a+b(n-2)≠0

Note that every manifold $(M^n, g)(n > 3)$ of constant curvature is an Einstein manifold. Thus we have

2.9) S(Y,Z)=$\frac{r}{n}g(Y,Z)$

and hence

2.10) QY=$\frac{r}{n}Y$

Further from 2.8) we see that

2.11) $r = -n(n-1)$ ,

Using 2.8), 2.9),2.10) and 2.11),we get from 1.10)

$$\tilde{C}(X,Y)Z{=}a\{g(X,Z)Y - g(Y,Z)X\} + b\{\frac{r}{n}g(Y,Z)X - \frac{r}{n}g(X,Z)Y + g(Y,Z)\frac{r}{n}X - g(X,Z)\frac{r}{n}Y\}$$

$$-\frac{r}{n}(\frac{a}{n-1} + 2b)\{g(Y,Z)X - g(X,Z)Y\}$$

$$=\{g(Y,Z)X - g(X,Z)Y\}(1 + \frac{r}{n(n-1)})$$

Finally using 2.11) , we get from above

$\tilde{C}(X,Y)Z{=}0$

Hence the manifold is quasi conformally flat. Thus we state

**Theorem 2:** A SP-Sasakian manifold $(M^n, g)(n > 3)$ with constant curvature is quasi conformally flat.

## III. QUASI CONFORMALLY AT SP-SASAKIAN MANIFOLD WITH R(X,Y).S=0

From 1.12) we find that

3.1) $S(R(X,Y)Z,W)+S(Z,R(X,Y)W)=0$

Taking Y=Z=$\xi$ in 3.1) and using 1.1), 1.7) ,1.8) we get

3.2) $-\frac{b}{a}\{-(n-1)S(X,W)+(n-1)\eta(X)S(\xi,W)+S(QX,W)+(n-1)\eta(X)S(\xi,W)$
$-(n-1)\eta(W)S(X,\xi)-S(X,W)S(\xi,\xi)+\eta(W)S(\xi,QX)+g(X,W)(n-1)S(\xi,\xi)\}$
$+\frac{r}{an}(\frac{a}{n-1}+2b)\{S(X,W)-\eta(X)S(\xi,W)+\eta(W)S(\xi,X)-g(X,W)S(\xi,\xi)\}=0$

or,3.3) $-\frac{b}{a}\{S(QX,W)-(n-1)^2\eta(X)\eta(W)-(n-1)\eta(W)\eta(QX)-(n-1)^2g(X,W)\}$

$$+\frac{r}{an}(\frac{a}{n-1}+2b)\{S(X,W)+(n-1)g(X,W)\}=0$$

Let $\lambda$ be any eigen value of the endomorphism Q corresponding to an eigen vector X.Then

$$QX=\lambda X$$

Hence 3.3) reduces to

$-\frac{b}{a}\{\lambda S(X,W)-(n-1)^2\eta(X)\eta(W)-\lambda(n-1)\eta(W)\eta(X)-(n-1)^2g(X,W)\}$

$$+\frac{r}{an}(\frac{a}{n-1}+2b)\{S(X,W)+(n-1)g(X,W)\}=0$$

Or, $-\frac{b}{a}\{g(\lambda X,W)-(n-1)^2\eta(X)\eta(W)-\lambda(n-1)\eta(W)\eta(X)-(n-1)^2g(X,W)\}$

$$+\frac{r}{an}(\frac{a}{n-1}+2b)\{g(\lambda X,W)+(n-1)g(X,W)\}=0$$

Taking W=$\xi$ , we get from above

$\lambda^2-2(n-1)^2-\lambda(n-1)-rp\{\lambda+(n-1)\}=0$ , as $\eta(X)\neq 0$,where

3.4) $p=\frac{1}{bn}(\frac{a}{n-1}+2b)$

Or,$\lambda^2-\{rp+(n-1)\}\lambda+\{-2(n-1)^2-rp(n-1)\}=0$

If$\lambda_1,\lambda_2$ are the non-null solutions of the above equation, then

3.5) $\lambda_1+\lambda_2=rp+(n-1)$, $\lambda_1\lambda_2=-2(n-1)^2-rp(n-1)$

Or, $\lambda_1-\lambda_2=\pm\{3(n-1)+rp\}$

When $\lambda_1-\lambda_2=3(n-1)+rp$ , then

$\lambda_1=2(n-1)+rp$ and $\lambda_2=-(n-1)$

When $\lambda_1-\lambda_2=-\{3(n-1)+rp\}$ ,then

$\lambda_1=-(n-1)$ and $\lambda_2=2(n-1)+rp$

On taking X= U= W , we get from 2.2)

$\frac{a}{b}g(R(W,Y)Z,W) = -\{S(Y,Z)g(W,W) - S(W,Z)g(Y,W) + g(Y,Z)S(W,W) - g(W,Z)S(Y,W)\}$

$$+\frac{r}{an}(\frac{a}{n-1} + 2b)\{g(Y,Z)g(W,W) - g(W,Z)g(Y,W)\}$$

Let us put $W = e_i$ where{ $e_i$: i=1,2 ....,n}is the set of orthonormal basis of tangent space at each point of M and summing for i=1,...,n we get

$\frac{a}{b}S(Y,Z) = -\{S(Y,Z).n - S(Y,Z) + rg(Y,Z) - S(Y,Z)\} + \frac{r}{an}(\frac{a}{n-1} + 2b)\{g(Y,Z)n - g(Y,Z)\}$

Or, $\frac{a+b(n-2)}{b}S(Y,Z) = \{-r + \frac{r.a}{bn} + \frac{r.2(n-1)}{n}\}g(Y,Z)$

Or, $S(Y,Z) = \frac{r}{n}g(Y,Z)$, Provided $a + b(n-2) \neq 0$

Thus we state

**Theorem 3:** A quasi-Conformally flat SP-Sasakian manifold with R(X,Y).S=0 reduces to an Einstein manifold provided a+b(n-2)≠ 0, where the symmetric endomorphism Q of the tangent space corresponding to the Ricci tensor S has two different non-zero eigen values.

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