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Counting the Conjugacy Classes of Finite Groups from the Centralizer

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ABSTRACT : *The concept of conjugacy class plays a central role in group and representation theory. In particular, the number and size of the conjugacy classes. The abelian case is trivial. However, in the non abelian case the number of conjugacy classes is less than the order of the group. Counting the centralizers in finite groups has been achieved by Sarah and Gary while Abdollahi, Amiri and Hassanabada have worked on groups with specific number of centralizers. In this paper we count the conjugacy classes for finite non abelian groups of prime power order using the centralizer. We derived some schemes in order to achieve this. These schemes give the upper bound on the conjugacy classes for finite non abelian groups of order p^w where w is a natural number and p is a fixed prime 2. The tool used here is the class equation.*

KEY WORDS**:** *centre, conjugacy classes, centralizer, nonabelian, class equation*

I. INTRODUCTION

Definition 1.1

A group G with the property that ab = ba for some pair of elements $a,b \in G$ is said to be a commutative or abelian group. A group in which there exist a pair of elements $a, b \in G$ endowed with the property that $ab \neq ba$ is called a non abelian or noncommutative group.

Definition 1.2

Let G be a group and $H < G$. For $q \in G$ the subset $Hq = \{hq:h \in H\}$ of G is called a right coset of H in G. Distinct right cosets of H in G form a partition of G. That is every element of G is precisely in one of them. Left coset is similarly defined. If G is commutative we just talk of coset of H. The number of distinct right cosets of H in G is written as $|G:H|$ and called the index of H in G. If G is finite so is H and G is partitioned into |G:H| cosets each of order |H| and we write:

$$|G:H| = \frac{|G|}{|H|}$$

We note that |H| and |G:H| divide |G|.

In the next definition, we have that a group consists of smaller groups and we give an analogous definition of subsets in groups.

Definition 1.3

A non empty subset N of a group G is said to be a subgroup of G written $N \leq G$, if N is a group under the operation inherited from G. If $N \neq G$, then N is a proper subgroup of G. If H is a non empty subset of G, then $H \leq G$ if and only if $xy^{-1} \in H$ whenever $x, y \in H$.

Any group of even order contains an element of order 2. A subgroup N of G such that every left coset is a right coset and vice versa is called a normal subgroup of G. That is $Nx = xN$ or $x^{-1}Nx \leq N$ and we write $N \triangleleft G$. A normal subgroup is characterized by the fact that it does not possess any conjugate subgroup apart from itself. That is $aH = Ha$ or $a^{-1}Ha = H$ for all a in G. If G is abelian then every subgroup of G is abelian.

Definition 1.4

The number of elements in a group G written |G| is called the order or cardinality of the group. If G is finite of order n we have $|G| = n$ otherwise $|G| = \infty$ if G has infinite order.

The least number n if it exists such that $a^n = 1$for a in G is called the order of a and we write $o(a) = n$. That is $o(a) = \min\{n > 0 : a^n = 1\}$. If no such n exists then $o(a) = \infty$. In the latter we say that powers of a are distinct but not all are distinct in the former. An element of order two is said to be an involution.

Lemma 1.5

Any group of even order contains an element of order 2. That is for $g \in G$ with $g \neq 1$ then $g^2 = 1$. In fact there are an odd number of such elements which are called involutions.

Proof

Since G has an even order then $|G| = 2m$. We note that $g^2 = 1$ if and only if $g = g^{-1}$, (1.4). We pair the non identity elements with their inverses and there are 2m - 1 of such elements. There is at least one g in G such that $g = g^{-1}$. This gives subset $\{g, g^{-1}\}$. By definition g is an involution and hence of order two.

A consequence of the decomposition in 1.2 is that if H is a subgroup of G then, $|G| = |G:H||H|$.

This naturally leads to an important theorem in group theory: The Lagrange's Theorem which is next.

Theorem 1.6

If G is a finite group and H is a subgroup of G then the order of H divides the order of G.

Proof

By 1.2 we have that the right cosets of H form a partition of G. Thus each element of G belongs to at least one right coset of H in G and no element can belong to two distinct right cosets of H in G. Therefore every element of G belongs to exactly one right coset of H. Moreover each right coset of H in G contains $|H|$ elements. Therefore if the number of right cosets of H in G is n, then $|G| = n|H|$. Hence the order of H divides the order of G.

Remark 1.7

Lagrange's Theorem greatly simplifies the problem of determining all the subgroups of a finite group. The converse of Lagrange's theorem is not true in general. That is if d is a divisor of the order of a finite group G, then it does not necessarily follow that G has a subgroup of order d. If d is a power of a prime number then the converse holds.

Definition 1.8

Let $a, q \in G$. Then a is conjugate to q in G if there exists an element $g \in G$ such that $q = g^{-1}ag$. The set of all elements of G that are conjugate to a in G is called the conjugacy class of a in G which we denote by C(a). And as such:

$$C(a) = \{g^{-1}ag : g \in G\}$$

We note that C(a) is a subgroup of G and by 1.6 its order divides that of G. Subgroups belonging to the same conjugacy class are conjugates. Such subgroups are isomorphic. The converse does not hold in general as we have in the case of abelian groups where two isomorphic subgroups may not be conjugates. However conjugate elements lie in the same conjugacy class and have the same order.

Remark 1.9 .

From Herstein (1964) conjugacy class induces a decomposition of G into disjoint equivalence classes (conjugate classes). This is a concept that is important in the theory of group representation and group characters. Again the character of a representation group is intimately tied with the conjugacy class of the group.

Definition 1.10

The centre Z(G) of a group G is the set of all elements z in G that commute with every element q in G. We write:

$$Z(G) = \{z \in G : zq = qz, \text{ for all } q \in G\}$$

and note that $Z(G)$ is a commutative normal subgroup of G and G modulo its centre $Z(G)$ is isomorphic to the inner automorphism, $inn(G)$ of G. If $Z(G) = \{1\}$ where 1 is the identity element of G, then G is said to have a trivial centre. The centre of a group G is its subgroup of largest order that commute with every element in the group. The divisors of $|G|$ reveal a lot about the order of $Z(G)$ and the conjugacy classes of G. If N is a normal subgroup of G such that $|N| = 2$, then $N \subseteq Z(G)$.We have the properties of the subgroups of the centre of the group G from Louis (1975) as follows .

Proposition 1.11
If H is a subgroup of $Z(G)$, the centre of the group G, then H is a normal subgroup of G. In particular $Z(G)$ is normal in G.

Proof
Since every $h \in H$ commutes with all elements in G we have that:

$$
\begin{aligned}
x^{-1}hx &= x^{-1}\{h : h\in H\}x \\
&=\{x^{-1}Hx : h\in H\} \\
&=\{x^{-1}xh : h\in H\} \\
&=\{h : h\in H\} \\
&= H
\end{aligned}
$$

And H is normal in G.

Proposition 1.12
If a, q are elements of G then:

(i) either $C(a) = C(q)$ or $C(a) \cap C(q) = \varnothing$;

(ii) or if a is a conjugate of q in G, then a^c is conjugate to q^c in G for every integer c and a and q have the same order.

Proof
(i) Suppose $C(a) \cap C(q) \neq \varnothing$ and let $x \in C(a) \cap C(q)$ then there exist $u.v \in G$. So that $x = u^{-1}au = v^{-1}qv$. Hence $a = uv^{-1}qvu^{-1} = g^{-1}qg$ with $g = vu^{-1}$ so

$$
\begin{aligned}
m \in C(a) \quad &\Rightarrow m = n^{-1}an \quad (m, n \in G) \\
&\Rightarrow m = n^{-1}g^{-1}qgn \\
&\Rightarrow m = d^{-1}qd, \quad d = gn \\
&\Rightarrow m \in C(q)
\end{aligned}
$$

And we have that $C(a) \subseteq C(q)$.
Similarly, using $q = g^{-1}a\,g$, $C(q) \subseteq C(a)$. Hence $C(a) = C(q)$

(ii) Observe that for $m, n \in G$, we have

$$u^{-1}mnu = (u^{-1}mu)(u^{-1}nu)$$

hence

$$u^{-1}a^{c}u = (u^{-1}au)^{c}.$$

Suppose that a is conjugate to q in G, so that $q = u^{-1}au$ for some u in G. Then

$$q^{c} = u^{-1}a^{c}u$$

and therefore a^c is conjugate to q^c in G. Let a have order w. That is $a^w = 1$. Then

$$q^{w} = u^{-1}a^{w}u = 1$$

and for

$$0 < r < w, q^i = u^{-1}a^i u \neq 1$$

So q also has order w.
Next we define an important concept and relate it to the conjugacy class.

Definition 1.13
The centralizer $C_G(q)$ of an element q in G is the set of all elements $g \in G$ that commute with q. That is:

$$C_G(q) = \{g \in G : gq = qg, \text{ for some } q \in G\}.$$

This is a subgroup of G and the index of $C_G(q)$ in G is the size of the conjugacy class C(q) of q in G. That is

$$|C(q)| = |G : C_G(q)|.$$

In particular |C(q)| divides |G|. If q is a central element $q \in Z(G)$ then |C(q)| = 1 and $q^{-1}gq = q$. So that $C_G(q) = G$.

The centralizer $C_G(q)$ of q in G is a subgroup of G but not a normal subgroup in general. Consequently the quotient of G by $C_G(q)$ is not a group

Next is the corollary to1.12 as can be seen from James and Martin (2001)

Corollary 1.14
If G is a finite group, then:

(i) every group is a union of its conjugacy classes and distinct conjugacy classes are disjoint;
(ii) conjugacy class is an equivalence relation where the equivalence classes are the conjugacy classes.

A relationship between the centre of G and the centralizer of the elements of G is given by:

Lemma 1.15
The centre Z(G) of a group G is the intersection of the centralizers $C_G(a)$ of elements a in G.

Definition 1.16
If $a \in G$, then $N_G(a)$ is the normalizer of a in G. It comprises of precisely the set of those elements in G which commute with a. It is a subgroup of G.

Herstein (1964) has it that if G is a finite group then the number of elements conjugate to a in G is the index of the normalizer of a in G. The conjugacy classes of a group are disjoint and their union form the group.

Remark 1.17
Let G be a group and h, g be elements of G. If the conjugacy classes of g and h overlap then the conjugacy classes are equal. The number of distinct or non-equivalent conjugacy classes is called the class number of the group G. In the symmetric group on n objects, each conjugacy class belongs to exactly one partition of n. The number of such conjugacy classes is equal to the number of integer partitions of n.
The next theorem presents the class equation for finite groups whose proof follows readily from 1.13

Theorem 1.18
Let G be a finite group then

$$|G| = \sum |G: C_G(q_i)| \qquad (i),$$

where the sum runs over the elements from each conjugacy class of G.
We note that from 1.13, equation (i) becomes

$$|G| = |Z(G)| + \sum |G: C_G(q_i)| \qquad (ii)$$

Here the sum in (ii) runs over q_i from each conjugacy class such that q_i is not an element of Z(G). equation (ii) above we have:

$$|G| = |Z(G)| + \sum|C(q_i)| \qquad (iii)$$

Remark 1.19

In the abelian environment, the sum in equation (iii) of 1.18 is zero. Consequently, the class equation is relevant only when we are in the non abelian environment. The fact that each element of Z(G) forms a conjugacy class containing just itself gives rise to the class equation.

Just as there are only n finite number of groups up to isomorphism with a given size, we also have that there is a finite number of groups up to isomorphism with a given number of conjgacy classes. Hence we have:

Theorem 1.20

The size of a finite group can be bounded above from knowing the number of its conjugacy classes.

Proof

When there is only one conjugacy class the group is trvial. Now fix a positive integer 1<k and let G be a finite group with k conjugacy classes represented by $g_1, g_2, ..., g_k$ including the g_i in the centre. From 1.18 (i) we another form of the class equation as:

$$|G| = \sum_{i=1}^{k} \frac{|G|}{|C_G(g_i)|}$$

dividing by |G| we have

$$1 = \frac{1}{n_1} + \frac{1}{n_2} + ... + \frac{1}{n_k} \qquad 1.20\ (i)$$

Where $n_i = |C_G(g_i)|$. We note here that each n_i exceeds 1 when G is not trivial and $n_1 \le n_2 \le ... \le n_k$. Then (i) imply that:

$$1 \le \frac{k}{n_1}$$

So $$n_1 \le k \qquad 1.20\ (ii)$$

Then using

$$n_i \ge n_2 \quad i \ge 2$$

$$1 \le \frac{1}{n_1} + \frac{k-1}{n_2}$$

Thus $$1 - \frac{1}{n_1} \le \frac{(k-1)}{n_2}$$ *, so*

$$n_2 \le \frac{k-1}{1 - 1/n_1} \qquad 1.20\ (iii)$$

By induction,

$$n_m \le \frac{k+1-m}{1 - (\frac{1}{n_1} + ... + \frac{1}{n_{m-1}})}, \qquad m \ge 2 \qquad 1.20\ (iv)$$

Since (ii) bounds n_1 by k and (iv) bounds each of $n_2, ..., n_k$, in terms of earlier n_1's, there are only a finite number of such k-tuples. The ones which satisfy (i) can be tabulated. The largest value of n_k

is |G|, since 1 has centralizer G, so the solution with the largest value for n_k *gives an upper bound on the size of a finite group with k conjugacy classes.*

Lemma 1.21
Let G be a group of order p^n*, with* $n \geq 1$ *then: If* $\{1\} \neq H \triangleleft G$*, we have that* $H \cap Z(G) \neq \{1\}$*. In particular* $Z(G) \neq \{1\}$*;*

Proof
Since $|G| = p^n$ *,* $H \triangleleft G$*. From 1.6 we have that |H| divides the order of G. This implies that |H| is a power of p. Furthermore,* $Z(G) \leq G$ *and given that* $Z(G) \neq \{1\}$*, we have that p divides the order of G. Now p also divides the orders of H and Z(G). Therefore* $H \cap Z(G) \neq \{1\}$ *and* $Z(G) \neq \{1\}$*.*

James and Martin (2001) proved the next Lemma.

Lemma 1.22
Let G be a group of order p^n *with* $1 \leq i \leq 4$*. Then G contains an abelian subgroup of index p.*
For the next definition see Sarah and Gary (1991) and Abdollahi (2007).

Definition 1.23
(i) We define $cent(G) = \{C_G(g) | g \in G\}$ *to be the distinct centralizers of the elements g in G. A group is called n – centralizer if the number of distinct centralizers in G is n. That is* $|cent(G)| = n$*. If* $|cent(G/Z(G))| = |cent(G)| = n$*, we call G a primitive n-centralizer, where n>0. There is only one centralizer in an abelian group.*

Definition 1.24
A subgroup N of a group G is said to be a proper centralizer of G if N is equal to the centralizer of an element g of G such that g is not in the centre of G. That is a subgroup N of G is called a proper centralizer of G if $N = C_G(g)$ *for some* $g \in G - Z(G)$*.*
We note that if G is a finite group, then G is the union of its proper centralizers cent(G). Furthermore, G is non abelian if and only if $|cent(G)| \geq 4$*.*
Abdollahi (2007) proved the next theorem

Theorem 1.25
The number of centralizers in a group G is four if and only if G modulo its centre is isomorphic to the Klein four group.

From Cody (2010) we have a theorem on the size of the conjugacy class of an element of G as follows.

Theorem 1.26
Let G be a finite group and $q \in G$*, then the conjugacy class C(q) of q in G is given by:*

$$|C(q)| = |G:C_G(q)| = |G|/|C_G(q)|.$$

Proof
Consider the function φ *that sends the coset* $xC_G(q)$ *to the conjugate* xqx^{-1} *of q. A routine calculation shows that* φ *is well defined, and is one to one and it maps the set of left cosets onto the conjugacy class of q. Thus the number of conjugates of q is the index of the centralizer of q.*

We note here that as a consequence of this theorem, if a, q are in the same conjugacy class then, $|C_G(q)| = |C_G(a)|$*. So if* $C(q) = \{q_1, q_2, ..., q_k\}$*, then*

$$\begin{aligned} \sum |C_G(q_i)| &= k|C_G(q)| \\ &= |G:C_G(q)||\, C_G(q\, | \\ &= |G|. \end{aligned}$$

The sum of the centralizers of all elements of G can be separated into sums of the centralizers of all the elements from each conjugacy class of G. That is if A_i are conjugacy classes of G, then we have:

$$\sum_{b\in G}|C_G(b)| = \sum_{x\in A_1}|C_G(x)| + \sum_{y\in A_2} C_G(y) + ... + \sum_{z\in A_m}|C_G(z)|$$

Therefore if we choose one element from each conjugacy class say b_i such that $1 \le i \le m$, then we have that:

$$\sum_{b\in G}|C_G(b)| = \sum_{i=1}^{m}|G : C_G(b_i)||C_G(b_i)|$$

$$= \sum_{i=1}^{m}|G| = m|G|$$

An immediate consequence is that: $|C_G(q)| = |C_G(a)|$ if a and q are in the same conjugacy class. For
$C(q) = \{q_i\, ,\ 1 \le i \le k\}$,
we have:

$$\sum |C(q)| = |G: C_G(q_i)||C_G(q_i)| = |G|.$$

Next we relate normal subgroup to conjugacy class:

Proposition 1.27
Let $N \le G$, then $N \lhd G$ if and only if N is the union of conjugacy classes of G.

Proof
If N is the union of the conjugacy classes of G, then for $n \in N$, $q \in G$ we have $q^{-1}nq \in N$. So $q^{-1}Nq \le N$. Conversely if $N \lhd G$ then for all $n \in N$, $q \in G$ we have that $q^{-1}nq \in N$. This implies that $C(n) \le G$ and so $N = \bigcup_{n\in N} C(n)$. Hence the result.
Mark (2011) proves the next theorem.

Theorem 1.28
If a finite group G has a centre Z(G) and G/Z(G) is cyclic then G is abelian.

From 1.6 we have:

Corollary 1.29
The order of an element a in G divides the order of G since <a> is a subgroup of G generated by a.

Corollary 1.30
For a finite non abelian group G and any element q in G:
(i) $|C_G(q)| = |Z(G)||C_G(q) : Z(G)|$ and $|G| = |C_G(q)||G: C_G(q)|$.
(ii) Equivalently $|Z(G)| = |C_G(q)|/(|C_G(q) : Z(G)|)$ and $|C_G(q)| = |G|/|G : C_G(q)|$.
Where $|C_G(q) : Z(G)| \ge 2$ and $|G : C_G(q)| \ge 2$, since G is finite and non abelian.
From Houshang and Hamid (2009) we have the next proposition which is an important property of p - groups and a consequence of 1.18.

Proposition 1.31
If the order of a finite group G is a power of a prime p then G has a non trivial centre. Equivalently the centre of a p - group contain more than one element.

Proof
Let G be the union between its centre and the conjugacy classes say J_i of size greater than 1.
Then from equation (iii) of 1.18

$$|G| = |Z(G)| + \sum|C(J_i)|$$

Each conjugacy class J_i has size of a power w say of prime p such that $w \ge 1$. In this case $w = 0$ for the conjugacy classes whose elements are central elements. Since each conjugacy class J_i has size a

power of p then $|J_i|$ is divisible by p. Furthermore as p divides $|G|$, it follows that p also divides $|Z(G)|$. Accordingly Z(G) is non- trivial.
Observe from 1.31 that there are elements of G other than the identity that commute with every element of G.

Mann (2011) proved:

Theorem 1.32
Let N be a normal abelian subgroup of a finite group G. Let also z be an element of N and x an element of G such that x is not in the centre of G. Then the conjugacy class of [x, z] has size smaller than that of the class of x.
What follow is an important theorem as in Cody (2010) and Jelten and Momoh (2014)

Theorem 1.33
If G is a finite non abelian group, then the maximum possible order of the centre of G is ¼$|G|$. That is, $|Z(G)| \leq 1/4|G|$.

Proof
Let $z \in Z(G)$. Since G is non abelian, $Z(G) \neq G$. Thus there exists an element $q \in G$ such that q is not in the centre. This implies that $C_G(q) \neq G$ and $C_G(q) \neq Z(G)$. Since $z \in Z(G)$ every element in G commute with z, so qz = zq. It follows that $z \in C_G(q)$. As $q \in C_G(q)$, we have that Z(G) is a proper subset of $C_G(q)$. Since a group that is a subset of a subgroup under the same operation is itself a subgroup of the subgroup, we find that Z(G) is a proper subgroup of $C_G(q)$. By 1.6 and 1.30, it follows that:

$$|Z(G)| \leq 1/2|C_G(q)|.$$

Now, since we assumed $C_G(q) \neq G$, then $C_G(q)$ is a proper subset of G. Therefore by 1.6 and the fact that the centralizer of any group element is a subgroup of G, we find that $|C_G(q)| \leq 1/2|G|$.That is:

$$\begin{aligned} |Z(G)| &\leq 1/2|C_G(q)| \\ &\leq 1/4|G|. \end{aligned}$$

We relate the centralizer of an element to the size of a finite non abelian group G as in Cody (2011)

Lemma 1.35
Let G be a finite non abelian group and $t \in G$ such that $t \notin Z(G)$, then:

$$C_G(t) = |G|/2.$$

Proof
We have from Theorem 1.33 that $|Z(G)| = |G|/4$ so that the number of centralizers of t such that $C_G(t) = G$ is $|G|/4$. We claim that the remaining ¾ elements of G each has order equal to $|G|/2$, for by the theorem $|G : Z(G)| = 4$. Since t is an element of G such that $t \neq Z(G)$, we have that:

$$|G : Z(G)| = |G : C_G(t)||C_G(t) : Z(G)|.$$

From theorem 1.6 we also have that $C_G(t) \neq G$ and $C_G(t) \neq Z(G)$ as $t \neq Z(G)$ and $t \in C_G(t)$, so from 1.6 and 1.30,

$$|C_G(t) : Z(G)| = 2.$$

We conclude that $|G : C_G(t)| = 2$ implying that $|C_G(t)| = |G|/2$.

Remark 1.36
In an abelian group, Z(G) = G, $C_G(t) = G$ for all t in G. But, $C_G(t) < G$ if G is non abelian. In which case we have $Z(G) < G$. The number of the centralizers that are equal to G is $|Z(G)|$.

II. OUR RESULTS

In our results we develop two schemes for computing the number of conjugacy classes for finite groups using the centralizer. These results give the upper bound for the number of conjugacy classes as shown in our conclusion.

Theorem 2.1

Let G be a finite nonabelian group whose order is p^w. *Let the order of the centralizer of an element x be* p^r *where w and r are positive integers such that* $r < w$, *then* $|C| = \frac{1}{4}(2p^w + p^r)$, *where |C| is the number of conjugacy classes.*

Proof

From the class equation we have

$$|G| = |Z(G)| + \sum_{i=1+|Z(G)|}^{|C|-|Z(G)|} |G : C_G(x)|, \ x \notin Z(G) \quad \text{with } |C(x)| \geq 2.$$

So that $|G| \geq |Z(G)| + 2(|C| - |Z(G)|)$

$$|G| \geq \frac{1}{2} C_G(x) + 2|C| - 2|Z(G)|$$

$$|G| \geq \frac{1}{2} C_G(x) + 2|C| - |C_G(x)|$$

$$2|G| \geq C_G(x) + 4|C| - 2|C_G(x)|$$

$$2p^w + p^r \geq 4|C|$$

$$\frac{2p^w + p^r}{4} \geq |C|$$

$\frac{1}{4}(2p^w + p^r) \geq |C|$ *as required*

Theorem 2.2

Given that a finite group G is of prime power order with centre Z(G), then we count the number of conjugacy classes from the centralizer as follows: $|C| \leq \frac{1}{4}(3|G| - |C_G(x)|)$.

$$|G| = |Z(G)| + \sum_{i=1+|Z(G)|}^{|C|-|Z(G)|} |G : C_G(x_i)|, \ x \notin Z(G) \quad \text{with } |C(x)| \geq 2.$$

So that $|G| \geq |Z(G)| + 2(|C| - |Z(G)|)$

$$|G| \geq \frac{1}{2} C_G(x) + 2|C| - 2|Z(G)|$$

$$|G| \geq \frac{1}{2} C_G(x) + 2|C| - \frac{1}{2}|G|$$

$$2|G| \geq C_G(x) + 4|C| - |G|$$

$$3|G| \geq C_G(x) + 4|C|$$

$$\frac{3|G| - |C_G(x)|}{4} \geq |C|$$

$$\frac{1}{4}(3|G| - |C_G(x)|) \geq |C|$$

That is $|C| \leq \frac{1}{4}(3|G| - |C_G(x)|)$

III. CONCLUSION

We conclude by using the schemes in our results to obtained the upper bounds for the number of the conjugacy classes for some groups of order p^w *with* $3 \le w \le 6$. *The table shows that the schemes are consistent and hence reliable for use by other researchers in group theory.*

p^w	$\|G\|$	$1/4(2p^w + p^r)$	$1/4(3\|G\| - \|C_G(x)\|)$
w = 3	*8*	*5*	*5*
w = 4	*16*	*10*	*10*
w = 5	*32*	*20*	*20*
w = 6	*64*	*40*	*40*

So a group of order p^3 *will have at most five conjugacy classes while the highest number of conjugacy classes a group of order* p^5 *has is twenty.*

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