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Steady Flow in Pipes of Rectangular Cross-Section Through Porous Medium

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ABSTRACT : *In this paper we have investigated the steady flow in pipes of rectangular cross-section through porous medium. We have investigated the velocity, flux and vortex line.*

KEY WORDS: *Steady flow, rectangular cross-section, incompressible fluid and porous medium.*

NOMENCLATURE

u = velocity component along x – axis
v = velocity component along y – axis
w (x , y) = velocity in x-y plane
t = the time
ρ = the density of fluid
P = the fluid pressure
K= the thermal conductivity of the fluid
μ = Coefficient of viscosity
υ = Kinematic viscosity
Q = the volumetric flow
Ω_x = Vorticity component in x – direction
Ω_y = Vorticity component in y – direction
Ω_z = Vorticity component in z - direction

I. INTRODUCTION

We have investigated the steady flow in pipes of rectangular cross-section through porous medium. Attempts have been made by several researchers D. Chittibabu and D.R.V. Prasada Rao [1] Sort effect on convective flow of heat & mass transfer through a Porous medium in a horizontal wavy dilated channel with radiation. D.K. Das and U. Barman [2] Slow steady flow of a viscous incompressible fluid between two infinite co-axial circular cylinders with axial roughness. D.K. Das and U. Barman [3] to study the boundary layer for MHD stratified fluid through a Porous medium. Dong – Yong Shui, Grassia Pau and Brian Derby [4] oscillatory in Compressible fluid flow in Tapered Tube with a free surface in an inkjet print Head. P.G. Drazin [5] on the stability of Parallel flow of an incompressible fluid of variable density and viscosity. L. E. Ericken and L. T. Fant [6] heat and mass transfer on a moving continuous flat plate with suction or Injection Ind. A. T. Eswar and B. C. Bommiah [7] the affect of variable viscosity on Laminar flow due to a point shrik. M. A. Al-Nimr, M. Alkam and M. Hamdan [8] on forced convection in channels partially filled with porous substrates. M. A. Al-Nimr and M. Alkam [9] unsteady Non-Darcian forced convection analysis in an annulus partially filled with a porous material. M.A. Al-Nimr and M. Alkam [10] unsteady Non-Darcian fluid flow in parallel plates channels partially filled with porous material. R. A. Alpher [11] on forced convection in channels partially filled with porous substrates. In this paper we have investigated the velocity, flux and vortex line.

II. FORMULATION OF THE PROBLEM

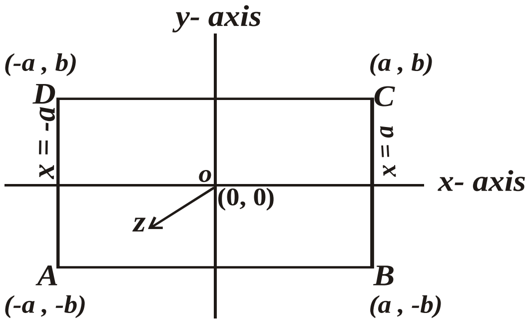
Let z-axis be taken the direction of flow along the axis of the pipe. Then u = 0, v = 0 for steady and incompressible fluid the velocity component is independent of z.
The equation of continuity.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$But \quad u=0,\ v=0,\ \frac{\partial w}{\partial z}=0 \Rightarrow w=w(x,y)\ldots\ldots(2)$$

i.e. w is independent of **z**

The Navier-Stokes equations of motion in the absence of body forces.



$$-\frac{\partial P}{\partial x} = 0 \;\ldots\ldots\ldots\ldots.(3)$$

$$-\frac{\partial P}{\partial y} = 0 \ldots\ldots\ldots(4)$$

$$= -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \frac{\mu w}{\rho K} = 0 \ldots\ldots\ldots\ldots\ldots.(5)$$

$$let \qquad \frac{1}{\rho K} = B^2$$

It is clear from (3) & (4) **P** is independent of **x** & **y** i.e. **p** is the Function of **z**

III. SOLUTION OF THE PROBLEM

$$p = p(z)\ , \qquad \frac{\partial p}{\partial z} = \frac{dp}{dz} = \text{Constant} = -\text{P}$$

$$\mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - B^2 w\right] = \frac{dp}{dz} \qquad \Rightarrow \qquad \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - B^2 w = -\frac{P}{\mu} \ldots\ldots\ldots\ldots\ldots.(6)$$

$$\left(D^2 + D'^2 - B^2\right) w = -\frac{P}{\mu}$$

$\therefore \quad C.F. = \sum a_n \, e^{h_n x + h'_n y}$ Where h_n & h'_n are related by $h_n^2 + h'^2_n - B^2 = 0$

$$\text{and} \quad P.I. = \frac{1}{D^2 + D'^2 - B^2}\left(-\frac{P}{\mu}\right) = \frac{P}{B^2 \mu}$$

$$w\,(x, y) = \sum_{n=1}^{\infty} a_n \, e^{h_n x + h'_n y} + \frac{1}{B^2 \mu} P \quad \text{Where} \quad h_n^2 + h'^2_n = B^2$$

Case - I: $w\,(x, y) = 0 \;\; at \;\; (a, b), \;\; w\,(x, y) = 0 \;\; at \;\; (a, -b)$

$$\sum_{n=1}^{\infty} a_n \, e^{h_n a + h'_n b} + \frac{P}{\mu B^2} = 0 \qquad and \qquad \sum_{n=1}^{\infty} a_n \, e^{h_n a - h'_n b} + \frac{P}{\mu B^2} = 0$$

$$\Rightarrow \quad -\frac{P}{\mu B^2} = \sum_{n=1}^{\infty} a_n \, e^{h_n a + h'_n b} \quad \ldots\ldots\ldots\ldots\ldots(a)$$

$$-\frac{P}{\mu B^2} = \sum_{n=1}^{\infty} a_n \, e^{h_n a - h'_n b} \quad \ldots\ldots\ldots\ldots\ldots(b)$$

$$h'_n = 0 \qquad \Rightarrow \qquad h_n = -B$$

$$\Rightarrow \quad -\frac{P}{\mu B^2} = e^{-aB}\sum_{n=1}^{\infty} a_n \qquad \Rightarrow \sum_{n=1}^{\infty} a_n = e^{-\frac{P}{\mu B^2}} e^{-aB} \quad , \quad w_1(x, y) = -\frac{P}{\mu B^2} e^{aB} e^{-xB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2} e^{B(-x+a)} + \frac{P}{\mu B^2}$$

Case -II: $w(x, y) = 0 \quad at \quad (-a, b) \ \& \ (-a, -b)$

$$w_2(x, y) = -\frac{P}{\mu B^2} e^{aB} e^{xB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2} e^{B(x+a)} + \frac{P}{\mu B^2}$$

Case - III: $w(x, y) = 0 \quad at \quad (-a, b) \ \& \ (a, b)$

$$w_3(x, y) = -\frac{P}{\mu B^2} e^{bB} e^{-yB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2} e^{B(-y+b)} + \frac{P}{\mu B^2}$$

Case - IV: $w(x, y) = 0 \quad at \quad (-a, -b) \ \& \ (a, -b) \quad , \quad w_4(x, y) = -\frac{P}{\mu B^2} e^{B(y+b)} + \frac{P}{\mu B^2}$

$$w(x, y) = \frac{P}{\mu B^2}\left[1 - 2 e^{aB} Cosh\, xB - 2 e^{bB} Cosh\, yB\right] \ldots\ldots\ldots\ldots. (7)$$

In particular case: In the case of square i.e. $a = b$

$$w(x, y) = \frac{P}{\mu B^2}\left[1 - 2 e^{aB}\left(Cosh\, xB + Cosh\, yB\right)\right] \ldots\ldots\ldots\ldots (8)$$

Flux Q of the fluid over an area of rectangular cross-section if given by

$$Q = \int_{x=-a}^{a}\int_{y=-b}^{b} w(x, y)\, dx\, dy = \int_{-a}^{a}\int_{-b}^{b} \frac{P}{\mu B^2}\left\{1 - 2 e^{aB} Cosh\, xB - 2 e^{bB} Cosh\, yB\right\} dx\, dy$$

$$= \frac{2P}{\mu B^2}\int_{-a}^{a}\int_{0}^{b}\left\{1 - 2 e^{aB} Cosh\, xB - 2 e^{bB} Cosh\, yB\right\} dy\, dx = \frac{2P}{\mu B^2}\int_{-a}^{a}\left\{\left(1 - 2 e^{aB} Cosh\, xB\right)b - 2 e^{bB}\left(\frac{1}{B} Sinh\, bB\right)\right\} dx$$

$$= \frac{4P}{\mu B^2}\int_{0}^{a}\left\{b\left(1 - 2 e^{aB} Cosh\, xB\right) - \frac{2}{B} e^{bB} Sinh\, bB\right\} dx = \frac{4P}{\mu B^2}\left[b\left\{x - \frac{2}{B} e^{aB} Sinh\, xB\right\}_{0}^{a} - \frac{2a}{B} e^{bB} Sinh\, bB\right]$$

$$= \frac{4P}{\mu B^2}\left[b\left\{a - \frac{2}{B} e^{aB} Sinh\, aB\right\} - \frac{2a}{B} e^{bB} Sinh\, bB\right]$$

$$Q = \frac{4P}{\mu B^2}\left[ab - \frac{2}{B}\left(b\, e^{aB} Sinh\, aB + a\, e^{bB} Sinh\, bB\right)\right] \ldots\ldots\ldots\ldots (9)$$

In particular case: In the case of square $a = b$

$$Q = \frac{4P}{\mu B^2}\left[a^2 - \frac{4a}{B} e^{aB} Sinh\, aB\right] \ldots\ldots\ldots\ldots (10)$$

$$w(x,y) = \frac{P}{\mu B^2}\left[1 - 2\, e^{aB}\, Cosh\, xB - 2\, e^{bB}\, Cosh\, yB\right]$$

$$\bar{q} = ui + vj + wk \ = \frac{P}{\mu B^2}\left[1 - 2\, e^{aB}\, Cosh\, xB - 2\, e^{bB}\, Cosh\, yB\right]\hat{k}$$

Let $\Omega_x,\ \Omega_y\ \&\ \Omega_z$ **are vorticity components**

$$\Omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = \frac{P}{\mu B^2}\left[-2B\, e^{bB}\, Sinh\, yB\right] = -\frac{2P}{\mu B}\, e^{bB}\, Sinh\, yB$$

$$\Omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = -\frac{P}{\mu B^2}\left[-2B\ e^{aB}\, Sinh\, xB\right] = \frac{2P}{\mu B}\ e^{aB}\, Sinh\, xB\ ,\ \Omega_z = 0$$

The equation of vortex line: $\frac{dx}{\Omega_x} = \frac{dy}{\Omega_y} = \frac{dz}{\Omega_z}$

$$\Rightarrow \quad \frac{dx}{-\frac{2P}{\mu B}\, e^{bB}\, Sinh\, yB} = \frac{dy}{\frac{2P}{\mu B}\, e^{aB}\, Sinh\, xB} = \frac{dz}{0}$$

$$dz = 0 \quad \Rightarrow \quad z = B$$

$$Again\ \frac{dx}{-e^{bB}\, Sinh\, yB} = \frac{dy}{e^{aB}\, Sinh\, xB} \Rightarrow e^{aB}\int Sinh\, xB\, dx + e^{bB}\int Sinh\, yB\, dy = C_1$$

$$\frac{1}{B}\, e^{aB}\, Cosh\, xB + \frac{1}{B}\, e^{bB}\, Cosh\, yB = C_1 \quad , \quad e^{aB}\, Cosh\, xB + e^{bB}\, Cosh\, yB = C_1 B = A$$

$\therefore$ **The vortex lines:**

$$e^{aB}\, Cosh\, xB + e^{bB}\, Cosh\, yB = A \qquad \& \qquad Z = B \ \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Clearly the flow is Rotational in pipe.

In particular case: In the case of square $a = b$

$$e^{aB}\left[Cosh\, xB + Cosh\, yB\right] = A \qquad \& \qquad Z = B \ \ldots\ldots\ldots\ldots (12)$$

Table for velocity:

$$Let\ \ P = \frac{1}{4},\ \ \mu = .5,\ \ a = b = 1,\ are\ same,\ B = \frac{1}{\sqrt{\sigma K}}\ \ and\ \ (x, y)\ are\ change$$

Table- 1 (for velocity)

	(x, y)	**(.1, .1)**	**(.2, .3)**	**(.3, .4)**	**(.4, .5)**	**(.5, .6)**	**(.6, .7)**	**(.7, .8)**
$\frac{1}{\sqrt{\rho K}} = 1$	$w(x, y)$	**-4.964**	**-5.114**	**-5.28**	**-5.504**	**-5.788**	**-6.134**	**-6.547**
$\frac{1}{\sqrt{\rho K}} = \frac{1}{2}$	$w(x, y)$	**-11.21**	**-11.297**	**-11.396**	**-11.529**	**-11.696**	**-11.897**	**-12.133**
$\frac{1}{\sqrt{\rho K}} = \frac{1}{3}$	$w(x, y)$	**-20.635**	**-20.712**	**-20.796**	**-20.908**	**-21.048**	**-21.216**	**-21.414**
$\frac{1}{\sqrt{\rho K}} = \frac{1}{4}$	$w(x, y)$	**-33.102**	**-33.172**	**-33.249**	**-33.352**	**-33.481**	**-33.636**	**-33.816**
$\frac{1}{\sqrt{\rho K}} = \frac{1}{6}$	$w(x, y)$	**-67.07**	**-67.135**	**-67.21**	**-67.3**	**-67.419**	**-67.56**	**-67.73**

IV. CONCLUSION AND DISCUSSION

In this paper we have investigated the velocity by the table- 1 of equations (7) between velocity and point (x, y). It is clear that the velocity of fluid increases uniformly with negative sign in the interval $(.1, .1) \le (x, y) \le (.7, .8)$ at different values of $\frac{1}{\sqrt{\rho K}}$ again velocity increases uniformly in the interval $(.1, .1) \le (x, y) \le (.7, .8)$ when $\frac{1}{\sqrt{\rho K}}$ decreases from 1 to $\frac{1}{6}$. Negative sign of velocity shows that direction of flow is opposite to the direction of motion of fluid. We have investigated vortex lines and the volumetric flow of elliptic and circle by equations. (8), (9), (10), (11) and (12) respectively.

REFERENCES

[1]. Chittibabu D. And D. R. V. Prasada Rao, Sort effect on convective flow of heat & mass transfer through a Porous medium in a horizontal wavy dilated channel with radiation. Acta ciencia Indica, Vol. XXXIV M, No. 4, pp 2135, (2008).

[2]. Das D. K. and U. Barman, Slow steady flow of a viscous incompressible fluid between two infinite co-axial circular cylinders with axial roughness. Acta ciencia Indica, Vol. XXXIV M, No. 4, pp 1625, (2008).

[3]. Das D. K. and U. Barman, to study the boundary layer for MHD stratified fluid through a Porous medium. Acta ciencia Indica, Vol. XXXIV M, No. 4, pp 1739, (2008).

[4]. Dong –Y. S. & Paul Grassia and B. D., in Compressible fluid flow in Tapered Tube with a free surface in an inkjet print Head. J. of fluid Engineering, Vol. 127, pp 98-109, (2005).

[5]. Drazin P. G., on the stability of Parallel flow of an incompressible fluid of variable density and viscosity. Proc. Camb. Phill. Soc., Vol. 58, pp 646, (1962).

[6]. Ericken L. E., Fant L. T. & Fox V. G., heat and mass transfer on a moving continuous flat plate with suction or Injection Ind. Engg. Chem. Fundam, pp 5-19, (1966).

[7]. Eswar A. T. and Bommiah B. C., the affect of variable viscosity on Laminar flow due to a point shrik. Indian J. Pure appl. Math., Vol. 35(6), pp 811–815, (2004).

[8]. Al-Nimr M. A., Alkam M. & Hamdan M., on forced convection in channels partially filled with porous substrates. Heat and Mass Transfer, Vol. 38(4-5) pp 337-342, (2002).

[9]. Al-Nimr M. A., Alkam M. K., unsteady Non-Darcian forced convection analysis in an annulus partially filled with a porous material. ASME Heat Transfer, Vol. 119, pp 799-785, (1997).

[10]. Al-Nimr M. A. & Alkam M.K., unsteady Non-Darcian fluid flow in parallel plates channels partially filled with porous material. Heat Mass Transfer, Vol. 33, pp 315-318, (1998).

[11]. Alpher R. A., on forced convection in channels partially filled with porous substrates. Int. J. of Heat and Mass Transfer, Vol. 3, pp 108, (1961).