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Squares and Square Roots

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ABSTRACT: *In this article I am giving easy and simple methods/ (KHAS method-**K**amal **H**aldar's **A**ddition and **S**ubtraction method) to solve squares and square roots. After knowing these methods learner can solve the problems in a short way without using the number of steps, It is said that methods are better than tricks because it can bring creativity in the mind.*

KEYWORDS: *Patterns, Squares, Square Root,*

I. INTRODUCTION,

In the previous methods we have learnt that the square of 12 can be written as 12^2 or 12x12 or $(10+2)^2$ or $(7+5)^2$ etc. This can be solved by using multiplication rule or using identities or other tricks. Here are the methods by which squares and square roots of given numbers can be solved easily. These methods are easy and less time taking. We need to see/know different methods for any problem because we need to develop our mind or to increase our intelligence.

1.1 Squares,

We know some patterns and properties of square number regarding these methods are as follows-

Pattern-1. The squares of any number having 0(zero) of its unit digit place in two digit number like

10, 20, 30,…….., 90.

For example,

10^2=100 20^2=400 30^2=900 40^2=1600 50^2=2500

60^2=3600 70^2=4900 80^2=6400 90^2=8100

Pattern-2. The squares of any number having 5 of its unit digit place in one and two digit number like

5,15,25,35,45,55,65,75,85,95

For example,

0$5^2$=25	**0**x (**0**+1) = **0**x1=*0*	5^2=25	→*0*25
1$5^2$=225	**1**x (**1**+1) = **1**x2=*2*	5^2=25	→*2*25
2$5^2$= 625	**2**x (**2**+1) = **2**x3=*6*	5^2=25	→*6*25
3$5^2$=1225	**3**x (**3**+1) = **3**x4=*12*	5^2=25	→*12*25
4$5^2$=2025	……..		
5$5^2$=3025	……..		

6$5^2$=4225 ……..

7$5^2$=5625 ……

8$5^2$=7225 ……

9$5^2$=9025 **9**x (**9**+1) = **9**x10=*90* 5^2=25 →*90*25

•Now find the squares of some numbers by **Addition/Subtraction** method(KHAS method)

Illustrative Examples

Example 1. Find the squares of the following numbers by **Addition/Subtraction** method:

(i) 23 (ii) 37 (iii) 123 (iv) 672 (v) 3689 (vi) 8732

Solution (i) 23^2=? (By Addition Method) – starting from 20^2

		Steps:-
20^2 = 400	***20+21*=41**	(i) Add 1st square number and successor of it (20+21=**41**)
+ **41**		**41** is called the ***1st number*.**
21^2 = 441	21+22=**43**	(ii) Add **41** to 400(square of 20) we get the square of 21
+ **43**		that is 441.
22^2 = 484	22+23=**45**	(iii) Add 2nd square number and successor of it (21+22=**43**)
+ **45**		(iv) Add **43** to 441(square of 21) we get square of 22
23^2 = 529 (**Ans.**)		that is 484.
		(v) Add 3rd square number and successor of it (22+23=**45**)
		(iv) Add **45** to 484 (square of 22) we get square of 23
		that is 529.

41, 43, 45 are the odd numbers differ by 2 in ascending order

In this way we can get the squares of successive numbers

In short-	20^2= 400	**Note-**
	+**129**	***** At first we need to find the ***1st number*** (20+21=**41**)
	23^2= 529(**Ans.**)	* Others numbers are the odd numbers differ by 2 in ascending order like 43, 45, 47, 49, 51,……..
		* 41+43+45=**129** (addition of three numbers is added to square of 20 to get the square of 23.

Solution (i) 23^2=? (By Subtraction Method) – starting from 25^2

		Steps:-
25^2= 625		(i) Add first square number and predecessor of it (25+24=**49**)
- **49**	25+24= **49**	**49** called the ***1st number***
24^2= 576		(ii) Subtract **49** from 625(square of 25) we get the square of 24
- **47**	24+23= **47**	that is 576.
23^2= 529 (**Ans.**)		(iii) Add second square number and predecessor of it
		(24+23=47)
		(iv) Subtract **47** from 576 (square of 24) we get the square of 23
		that is 529.

In short- $25^2 = 625$
$\underline{-96}$
$23^2 = 529$ (**Ans.**)

Note-
***** At first we need to find the ***1st number*** (25+24=**49**)
* Other numbers are the odd numbers differ by 2 in descending order like 47, 45, 43, 41, 39,……..
* 49+47 = **96** (addition of two numbers is subtracted from the square of 25 to get the square of 23.
After 25^2 there are two steps backward 24^2 and 23^2 to get the value of 23^2

Solution (ii) 37^2=?
$\underline{37} \rightarrow$ **35** **OR** $\underline{37} \rightarrow$ **40**
$35^2 \rightarrow 37^2$

we can start from **35^2**
$35^2 = 1225$
$\underline{+ \mathbf{71}}$ 35+36=**71**
$36^2 = 1296$
$\underline{+ \mathbf{73}}$ 36+37=**73**
$37^2 = 1369$ (**Ans.**)

in short-
$35^2 = 1225$
$\underline{+ 144}$ **71**+**73**=144
$37^2 = 1369$ (**Ans.**)

Solution (ii) 37^2=?
$\underline{37} \rightarrow 40$
$40^2 \rightarrow 37^2$

also we can start from **40^2**
$40^2 = 1600$
$\underline{- \mathbf{79}}$ 40+39=**79**
$39^2 = 1521$
$\underline{- \mathbf{77}}$ 39+38=**77**
$38^2 = 1444$
$\underline{- \mathbf{75}}$ 38+37=**75**
$37^2 = 1369$ (**Ans.**)

in short-
$40^2 = 1600$
$\underline{- 231}$ **79+77+75=**231
$37^2 = 1369$(**Ans.**)

Solution (iii) 123^2 =?
$\underline{123} \rightarrow \underline{120} \rightarrow 12 \rightarrow$ **10** **OR** $\underline{123} \rightarrow \underline{120} \rightarrow 12 \rightarrow$ **15**

First change the unit's digit of 123 into 0 as 120
Then remove zero from 120 becomes 12, and it is nearest to 10 or15
We can start from **10^2** *or* **15^2**
Now we start to solve it from **10^2**

$10^2 \rightarrow 12^2 \rightarrow 120^2 \rightarrow 123^2$

$10^2 = 100$ 10+11=21 (21 is added to 100 to get 11^2=121 again
$\underline{+ \mathbf{44}}$ $\underline{+ 23}$ 23 is added to 121 to get 12^2=144)
$12^2 = 144$ **44**

$120^2 = 14400$ 120+121=241 (241 is the 1st number and 243and 245 are the
$\underline{+ \mathbf{729}}$ +243 odd numbers in ascending order up to 3 steps)
$123^2 = 15129$ (**Ans.**) $\underline{+245}$
729

Solution (iv) 672^2 =?

672 → 670 → **65** (we can start from **65**) **OR** 672→670→**70** (also we can start from **70**)

First change the unit's digit of 672 into 0 as 670
Then remove zero from 670 becomes 67, and it is nearest to 65or70

Now we start to solve it from **65^2**

65^2 → 67^2 → 670^2 → 672^2

65^2 = 4225 (6x7) and (**5x5**) = 4225 (by pattern 2) 65+66=131
+ **264** + 133
67^2 = 4489 **264**

670^2 = 448900 670+671= 1341
+ **2684** +1343
672^2= 451584 (**Ans.**) **2684**

Solution (v) 3689^2 =?

3689→ 3690 → 369→370 →37→ **40** **OR** 3689 → 3690 → 369→370 →37→ **35**

We can start from **40^2** *or* **35^2**

Now we start to solve it from **40^2**

40^2 → 37^2 → 370^2 → 369^2 → 3690^2 → 3689

40^2 = 1600 40+39 = 79
- **231** + 77
37^2 = 1369 + 75
231

370^2 = 136900 370+369 = **739**
- **739**
369^2 = 136161

3690^2 = 13616100 3690+3689 = **7379**
- **7379**
3689^2 = 13608721 **(Ans.)**

Solution (vi) 8732=?

8732 → 8730 → 873→870 →87→ **85** **OR** 8732 → 8730 →873→ 870→87 → **90**

Now we start to solve it from **85^2**

85^2 → 87^2 → 870^2 → 873^2 → 8730^2 → 8732^2

$85^2 = 7225$ **8**5^2= **72**25 (by pattern 2) 85+86=171

+ **344** + 173

$87^2 = 7569$ **344**

$870^2 = 756900$ 870+871= 1741

+ **5229** + 1743

$873^2 = 762129$ + 1745

5229

$8730^2 = 76212900$ 8730+8731= 17461

+ **34924** + 17463

$8732^2 = 76247824$ (**Ans.**) **34924**

1.2 Square Roots,

•Opposite (inverse) operation of square is called Square Root.

Example 2. Now find the square root of the given numbers

(i) 529 (ii) 1369 (iii) 451584 (iv) 76247824 (v) 3526418

Solution (i) √**5** 29 **=?**

(**5**29 will be the perfect square of two digit number)

5→ 2^2= 4 (4 should **less** and nearest to **5**), but 3^2 = 9 (9 is greater than **5**) then **Tens digit is 2**

20^2 = 400 (400 is nearest to **5** 29) and 30^2 = 900 (900 is far)

Now we can start to solve it from 20^2

20^2= 400

+41 20+21= **41**(1st number) (529 is a perfect square number between 20^2 and 30^2 then 20^2= 400 and 30^2= 900 in which ***5*** *29* ***is nearest to 400,*** so we can start solving from 20^2= 400)

21^2= 441

+43 21+22= **43**

22^2= 484

+45 22+23= **45**

23^2= 529

then **Ones digit is 3** (**41, 43, 45** are the odd numbers differ by 2 in ascending order)

Hence √**5** 29**=23** (**Ans.)**

Solution (ii) √**13** 69 **=?**

(**13** 69 will be the perfect square of two digit number)

13→ 3^2= 9 (9 should **less** and nearest to13), but 4^2 = 16 (16 is greater than **13**) then **Tens digit is 3**

30^2=900(Far) and 40^2 =1600 (*1600 is nearest to* ***13*** *69*)

Now we can start to solve it from 40^2 *also can start to solve it from 35^2*

40^2 = 1600
- **79** 40+39=**79**
39^2 = 1521
- **77** 39+38=**77**
38^2 = 1444
- **75** 38+37=**75**
37^2 = 1369

(**13** 69 is a perfect square number between 30^2 and 40^2 then 30^2= 900 and 40^2=1600 in which ***13*** *69* ***is nearest to 1600,*** so we start solving from 40^2= 1600)

then **Ones digit is 7**

(**79, 77, 75** are the odd numbers differ by 2 in descending order)

Hence √**13** 69 = **37** (**Ans.)**

Solution (iii) √**45** 15 **84** **=?**

(**45** 15 **84** will be the perfect square of three digit number)

45 → 6^2= 36(36 should **less** nearest to **45**), but 7^2= 49(49 is greater than **45**)
then **Hundreds digit is 6**

70^2 = 4900 It is nearest to **45** 15(four digits from left) and 65^2 = 4225 also 60^2 = 3600(Far)

Now we can start to solve it from 70^2 *also can start to solve it from 65^2*

70^2 = 4900
-411
67^2 = 4489 4489 is less and nearest to **45** 15

70+69= 139
+ 137
+ 135
411

65^2 = 4225
+ 264
67^2 = 4489

then **Tens digit is 7**

670^2 = 448900 670+671= **1341, 1343,……..**
+ 1341
671^2 = 450241
+ 1343
672^2 = 451584

Ones digit is 2

Hence √**45** 15 **84**= 672 **(Ans.)**

Solution (iv) √**76** 24 **78** 24 **=?**

(**76** 24 **78** 24 will be the perfect square of four digit number)

76 → 8^2= 64(36 should **less** nearest to **45**), but 9^2 =81(81 is greater than **76**)
then **Thousands digit is 8**

80^2 = 6400(Far) and 90^2 = 8100 (8100 is nearest to **76** 24 four digits from left)

Now we can start to solve it from 90^2

80^2=6400 and 90^2 = 8100
- 179
89^2 = 7921

90+89= (**179, 177, 175** are the odd numbers differ by 2 in descending order)

$$\begin{array}{rl} & \underline{\mathbf{-\ 177}} \\ 88^2 = & 7744 \\ & \underline{\mathbf{-\ 175}} \\ 87^2 = & 7569 \end{array}$$ (It is less and nearest to **76** 24)

then **Hundreds digit is 7**

$$\begin{array}{rl} 870^2 = & 756900 \\ & \underline{+\ \mathbf{1741}} \\ 871^2 = & 758641 \\ & \underline{+\ \mathbf{1743}} \\ 872^2 = & 760384 \\ & \underline{+\ \mathbf{1745}} \\ 873^2 = & 762129 \end{array}$$

870+871= (**1741, 1743, 1745,** are the odd numbers differ by 2 in ascending order)

(It is less than and nearest to**76** 24 **78**)

then **Tens digit is 3**

$$\begin{array}{rl} 8730^2 = & 76212900 \\ & \underline{+\ \mathbf{17461}} \\ 8731^2 = & 76230361 \\ & \underline{+\ \mathbf{17463}} \\ 8732^2 = & 76247824 \end{array}$$

8730+8731 = (**17461, 17463** are the odd numbers differ by 2 in ascending order)

then **Ones digit is 2**

Hence $\sqrt{}$**76** 24 **78** 24=8732 (**Ans.**)

Solution (v) $\sqrt{}$**3** 52 **64** 18 **=?** **OR** $\sqrt{}$**3** 52 **64** 18 **. 00** 00 **= ?**

3 → 1^2= 1(1 should **less** and nearest to **3**), but 2^2 =4(4 is greater than **3**)

then **Thousands digit is 1**

$1^2 = 1$ and $2^2 = 4$ (4 is nearest to **3)**

Now we can start to solve it from 20^2

$$\begin{array}{rl} 20^2 = & 400 \\ & \underline{\mathbf{-\ 39}} \\ 19^2 = & 361 \\ & \underline{\mathbf{-\ 37}} \\ 18^2 = & 324 \end{array}$$

20+19 = 39(**39, 37, 35** are the odd numbers differ by 2 in descending order)

(It is less and nearest to **3** 52)

then **Hundreds digit is 8**

$180^2 = 32400$ and $190^2 = 36100$ 36100 is nearest to **3** 52 **64** (Five digits from left)

$$\begin{array}{rl} & \underline{\mathbf{-\ 379}} \\ 189^2 = & 35721 \\ & \underline{\mathbf{-\ 377}} \\ 188^2 = & 35344 \\ & \underline{\mathbf{-\ 375}} \\ 187^2 = & 34969 \end{array}$$

190+189 = 379 (**379, 377, 375 ……**are differ by 2 in descending order)

(It is less than and nearest to **3** 52 **64**)

then **Tens digit is 7**

$1870^2 = 3496900$ and $1880^2 = 3534400$ 3534400 is nearest to **3** 52 **64** 18 (Seven digits from left)

$$\begin{array}{rl} & \underline{\mathbf{-\ 3759}} \\ 1879^2 = & 3530641 \\ & \underline{\mathbf{-\ 3757}} \\ 1878^2 = & 3526884 \\ & \underline{\mathbf{-\ 3755}} \end{array}$$

1880+1879 = **3759** (**3759, 3757, 3755 ……**are differ by 2 in descending order)

$1877^2 = 3523129$ (It is less than and nearest to **3** 52 **64** 18)

then **Ones digit is 7**

$18770^2 = 352312900$ and $18780^2 = 352688400$ It is nearest to **3** 52 **64** 18 **00**

\- **37559** 1880+1879 = **3759** (**3759, 3757, 3755 …**are

$18779^2 = 352650841$ differ by 2 in descending order)

\- **37557**

$18778^2 = 352613284$ (It is less than and nearest to **3** 52 **64** 18 **00**)

then **Tenth digit is 8**

$187780^2 = 35261328400$ and $187790^2 = 35265084100$ It is nearest to **3** 52 **64** 18 **00** 00

\- **375579** ← **375579 (1st no.) =** 187790+187789

$187789^2 = 35264708521$

\- **375577** **375577, 375575,….** are differ by 2

$187788^2 = 35264332944$ in descending order

\- **375575**

$187787^2 = 35263957369$

then **Hundredth digit is 7**

Hence √**3** 52 **64** 18 **. 00** 00 = 1877.87 app. (**Ans.**)

II. CONCLUSION

These methods are very useful for all types of students like brilliant, average or below average. Learner can feel free from the complication that only addition/subtraction methods are used. Decimal numbered questions can be solved by the same methods shown as above. More number of Steps may be appearing in the solution of the problem but after knowing these methods it can be solved by very short way and no need to write the other things except solutions and a little rough work.

REFERENCES

[1] Pattern-1, General calculations (we know)

[2] Pattern-2, Mathematics books are published in India (not any particular book)