A Study on Star Intuitionistic Sets

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ABSTRACT. The aim of this paper is to introduce a new type of Intuitionistic sets known as the star Intuitionistic sets and study some of its properties. 2000 Math. Subject Classification: 54C10, 54C08.

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I. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy sets were introduced and investigated by "Zadeh[11]" in 1965. For the rst time, the concept of a topological structures was generalized to fuzzy topological spaces[5] in 1968 and the concept of generalized Intuitionistic fuzzy sets was considered by K.Atanassov [2] in 1983. "Intuitionistic fuzzy topological space"were introduced by Coker in [7]. Intuitionistic sets in point set was also de ned by Coker[8] in 1996. In this paper, we de ne a new operator on intuitionistic sets, which results again an intuitionistic set which we call it as a star intuitionistic set. We also study some of their properties.

De nition 1.1. [9]

Let X be a non empty fixed set. Then the set A = < X, A1, A2 > Where A1 and A2 are subsets of X is called an intuitionistic set if A1 A2 = ø The set A1 is called the set of member of A, A2 is called the set of non member of A Here after let us represent the intuitionistic set as IS-sets.

De nition 1.2. [9]

(a) Let X and Y are two non empty fixed sets. Let A = < X, A1, A2 > and B = < Y, B1, B2 > be two IS sets defined on X and Y respectively. Then the image of A under f, denoted by f(A), is the IS in Y defined by f(A) = < Y, f(A1), f(A2) >, where f(A2) = (f(A2)c)c.

(b) If X and Y are two non empty fixed sets. Let A = < X, A1, A2 > and B = < Y, B1, B2 > be two IS sets defined on X and Y respectively. Then the preimage of B under f, denoted by f−1(B), is the IS in X defined by f−1(B) = < X, f−1(B1), f−1(B2) >.

De nition 1.3. [9]

An intuitionistic topology(IT for short) on a nonempty set X is a family τ of ISs in X satisfying the following axioms:

(T 1) ø, X ∈ τ

(T 2) G1 G2 ∈ τ for any G1, G2 ∈ τ.

(T 3) ∪ Gi ∈ τ for any arbitrary family {Gi : i ∈ J } ⊆ τ.
Definition 1.4. [9] Let $(X, \tau)$ be an ITS and $A = \langle X, A^1, A^2 \rangle$ be an IS in $X$. Then the interior and closure of $A$ are defined by
\[ \text{Cl}(A) = \cap \{K : K \text{ is an ICS in } X \text{ and } A \subseteq K\}. \]
\[ \text{int}(A) = \cup \{G : G \text{ is an IOS in } X \text{ and } G \subseteq A\}. \]

Definition 1.5. [8] Let $X$ be a nonempty set and $p \in X$ a fixed element in $X$. Then the IS $\bar{p} = \langle x, \{p\}, \{p\}^c \rangle$ is called an intuitionistic point/IP for short) in $X$.

IP’s in $X$ can sometimes be inconvenient when express an IS in $X$ in terms of IP’s. This situation will occur if $A = \langle X, A^1, A^2 \rangle$ and $p \notin A_1$. Therefore we shall define vanishing IP’s as follows:

Definition 1.6. [8] Let $X$ be a nonempty set and $p \in X$ a fixed element in $X$. Then the IS $p_\infty = \langle x, \phi, \{p\}^c \rangle$ is called a vanishing intuitionistic point (VIP for short) in $X$.

Definition 1.7. [8] Let $f : X \to Y$ be a function.

(a) Let $\bar{p}$ be an IP in $X$. Then the image of $\bar{p}$ under $f$, denote by $f(\bar{p})$, is defined by $f(\bar{p}) = \langle Y, \{q\}, \{q\}^c \rangle$, where $q = f(p)$ and

(b) Let $p_\infty$ be a VIP in $X$. Then the image of $p_\infty$ under $f$, denoted by $f(p_\infty)$, is defined by $f(p_\infty) = \langle Y, \phi, \{q\}^c \rangle$, where $q = f(p)$.

It is easy to see that $f(\bar{p})$ is indeed an IP in $Y$, namely $f(\bar{p}) = \bar{q}$ where $q = f(p)$, and it has exactly the same meaning of the image of an IS under the function $f$. $f(p_\infty)$ is also a VIP in $Y$, namely $f(p_\infty) = p_\infty$, where $q = f(p)$.

Definition 1.8. [9]

Let $X$ be a nonempty fixed set. Then the operators $\llbracket, \rrbracket, \ll , \lll$ are defined on an intuitionistic set as $\llbracket A \rrbracket = \langle X, A^1, (A^1)^c \rangle$ and $\ll, \lll A = \langle X, (A^2)^c, A^2 \rangle$.

Lemma 1.9. [9]

If $A = \langle X, A^1, A^2 \rangle$ is an IS set, then $\overline{A} = \langle X, A^2, A^1 \rangle$. 
Definition 1.10. [9] Let $(X, \tau)$ be a ITS.

(a) $\tau_1 = \{G^1 : \in X, G^1, G^2 \in \tau\}$ is a topological space on $X$. Similarly $\tau_2 = \{G^2 : \in X, G^1, G^2 \in \tau\}$ is a family of all closed sets of the topological space $\tau_2 = \{(G^2)^c : \in X, G^1, G^2 \in \tau\}$ on $X$.

(b) Since $G^1 \cap G^2 = \emptyset$ for each $G = \in X, G^1, G^2 \in \tau$, we obtain $G^1 \subseteq (G^2)^c$ and $G^2 \subseteq (G^1)^c$. Hence $(X, \tau_1, \tau_2)$ is a bitopological space.

II. STAR INTUITIONISTIC SETS

In this chapter, we define a new IS namely star intuitionistic set and studied some of their basic properties.

Definition 2.1. Let $X$ be a non empty fixed set and $A = \in X, A^1, A^2 \in \tau$ be an intuitionistic set. Then we define the star intuitionistic set $A^* = \in X, (A^2)^c \cap (A^1)^c >$, where $A^1$ and $A^2$ are the subsets of $X$.

Lemma 2.2. Let $X$ be a non empty set and $A = \in X, A^1, A^2 >$ be an intuitionistic set. Then $A^* = \in X, (A^2)^c \cap (A^1)^c >$ is also an intuitionistic set.

proof:

To Prove: $\in X, (A^2)^c \cap (A^1)^c >$ is an IS, we have to prove that $( (A^2)^c \cap (A^1)^c ) \cap ( (A^2) \cap (A^1)^c ) = \emptyset$, which is so obvious and so

$A^*$ is also an intuitionistic set.

Corollary 2.3. Let $X$ be a non empty set. Then $\bar{\phi}^* = \in X, \phi^c - X^c, \phi \cap X^c >$ and $\bar{X}^* = \in X, X \cap \phi^c, X^c - \phi^c >$ are also star intuitionistic set.

Theorem 2.4. Let $X$ be a non empty set with $A = \in X, A^1, A^2 \in \tau$ and $B = \in X, B^1, B^2 \in \tau$ be two given intuitionistic sets with $A^i, B^i (i = 1, 2)$ are subsets of $X$. If $A^* = \in X, (A^2)^c \cap (A^1)^c >$ and $B^* = \in X, (B^2)^c - (B^1)^c, (B^2) \cap (B^1)^c >$ are star intuitionistic sets on $X$, then $A \subseteq B$ implies $A^* \subseteq B^*$.

proof:

Given $A \subseteq B$. Then $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$

It is easy to prove that $(A^2)^c - (A^1)^c \subseteq (B^2)^c - (B^1)^c$ and $(B^2) \cap (B^1)^c \subseteq A^2 \cap (A^1)^c$. So, $A^* \subseteq B^*$. 

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Remark 2.5. \( A^* = B^* \) iff \( A^* \subseteq B^* \) and \( B^* \subseteq A^* \).

Corollary 2.6. We can also prove the equalities

(i) \( \overline{A^*} = \overline{(X, A_2^* - A_1^*), (A^2) \cap (A^1)c, A_2^* - A_1^*} \).

(ii) \( \cup A_i^* = \overline{(X, \cup A_i^2), (\cup A_i^1)c, (\cup A_i^2) \cap (\cup A_i^1)c} \).

(iii) \( \cap A_i^* = \overline{(X, (\cap A_i^2)c, (\cap A_i^1)c, (\cap A_i^2) \cap (\cap A_i^1)c} \).

(iv) \( A^* - B^* = A^* \cap B^c \).

and it is easy to show that each R.H.S is also a star intuitionistic sets.

Corollary 2.7. The operators \( [\cdot], \prec \) defined on an intuitionistic set can also be extended to star intuitionistic set as follows.

(i) \( \overline{A^*} = \overline{(X, (A^2)c - (A^1)c, (A^2)c - (A^1)c)c} \).

(ii) \( \prec A^* = \overline{(X, (A^2) \cap (A^1)c, ((A^2) \cap (A^1)c)c} \).

Here are some of the basic properties of the inclusion and complementation of star IS.

Corollary 2.8. Let \( A_i \) be IS’s in \( X \) where \( i \in J \), where \( J \) is an index set and \( A_i \) are corresponding star IS sets defined on \( X \) then

(a) \( A_i^* \subseteq B^* \) for each \( i \in J \Rightarrow \cup A_i^* \subseteq B^* \).

(b) \( B^* \subseteq A_i^* \) for each \( i \in J \Rightarrow B^* \subseteq \cup A_i^* \).

(c) \( \cup A_i^* = \cap A_i^* \); \( \cap A_i^* = \cup A_i^* \).

(d) \( A^* \subseteq B^* \Leftrightarrow B^* \subseteq A^* \).

(e) \( \overline{(A^*)} = A^* \).

(f) \( \overline{\phi^*} = \overline{X^*}, \overline{X^*} = \overline{\phi^*} \).

Now we shall define the image and preimage of star ISs. Let \( X, Y \) be two nonempty fixed sets and \( f: X \rightarrow Y \) be a function.

Let \( A \) and \( B \) be the IS sets on \( X \) and \( Y \) respectively.

Definition 2.9. (a) If \( B^* = \prec Y, (B^2)c - (B^1)c, B^2 \cap (B^1)c \) is a star IS in \( Y \), then the preimage of \( B \) under \( f \), denoted by \( f^{-1}(B) \), is the star IS in \( X \) defined by \( f^{-1}(B^*) = \prec X, f^{-1}((B^2)c - (B^1)c), f^{-1}(B^2 \cap (B^1)c) \).

(b) If \( A^* = \prec X, (A^2)c - (A^1)c, A^2 \cap (A^1)c \) is a star IS in \( X \), then the image of \( A \) under \( f \), denoted by \( f(A^*) \), is the star IS in \( X \) defined by \( f(A^*) = \prec Y, f((A^2)c - (A^1)c), f(A^2 \cap (A^1)c) \). Where \( f(A^2 \cap (A^1)c) = (f(A^2 \cap (A^1)c)c = Y - f(X - (A^2 \cap (A^1)c)) \).
Lemma 2.10. Let $A^* = \prec X, (A^0)^c - (A^1)^c, A^2 \cap (A^1)^c \succ$ is an Intuitionistic set. Then $A^2 \cap (A^1)^c \supseteq f^{-1}(f_+(A^2 \cap (A^1)^c))$.

proof:

$f^{-1}(f_-(A^2 \cap (A^1)^c)) = f^{-1}(Y - f(X - (A^2 \cap (A^1)^c)))$

$= f^{-1}(Y) - f^{-1}(f(X - (A^2 \cap (A^1)^c)))$

$\subseteq X - (X - (A^2 \cap (A^1)^c))$

$= A^2 \cap (A^1)^c$

$f^{-1}(f_-(A^2 \cap (A^1)^c)) \subseteq A^2 \cap (A^1)^c$

Theorem 2.11. Let $A_i^*(i \in J)$ be star IS sets corresponding to the IS sets $A_i$ in $X$ and $B_j^*(j \in k)$ be star IS's corresponding to the IS sets $B_j$ in $Y$, and $f : X \rightarrow Y$ be a function. Then

(a) $A_i^* \subseteq A_j^* \Rightarrow f(A_i^*) \subseteq f(A_j^*)$.

(b) $B_i^* \subseteq B_j^* \Rightarrow f^{-1}(B_i^*) \subseteq f^{-1}(B_j^*)$.

(c) $A^* \subseteq f^{-1}(f(A^*))$ and if $f$ is injective, then $A^* = f^{-1}(f(A^*))$.

(d) $f(f^{-1}(B^*)) \subseteq B^*$ and if $f$ is surjective, then $f(f^{-1}(B^*)) = B^*$.

(e) $f^{-1}(\cup B_i^*) = \cup f^{-1}(B_i^*)$,

(f) $f^{-1}(\cap B_i^*) = \cap f^{-1}(B_i^*)$.

(g) $f(\cup A_i^*) = \cup f(A_i^*)$.

(h) $f(\cap A_i^*) \subseteq \cap f(A_i^*)$, and if $f$ is injective, then $f(\cap A_i^*) = \cap f(A_i^*)$.

(i) $f^{-1}(\tilde{X}^*) = \tilde{X}^*$,

(j) $f^{-1}(\tilde{\phi}^*) = \tilde{\phi}^*$,

(k) $f(\tilde{\phi}^*) = \tilde{\phi}^*$,

(l) $f(\tilde{X}^*) = \tilde{Y}^*$, if $f$ is surjective.

(m) If $f$ is surjective, then $f(A^*) \subseteq f(A^*)$. If furthermore, $f$ is injective, then have $f(A^*) = f(A^*)$.

(n) $(f^{-1}(\overline{B^*})) = f^{-1}(\overline{B^*})$. 

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proof:

(a) Given $A_1^t \subseteq A_2^t$, where $A_1^t = <X, (A_1^2)^c - (A_1^1)^c, A_1^2 \cap (A_1^1)^c >$

$A_2^t = <X, (A_2^2)^c - (A_2^1)^c, A_2^2 \cap (A_2^1)^c >$

To Prove: $f(A_1^t) \subseteq f(A_2^t)$

By definition $f(A_1^t) = <Y, f((A_1^2)^c - (A_1^1)^c), f_-(A_1^2 \cap (A_1^1)^c) >$. Where $f_-(A_1^2 \cap (A_1^1)^c) = (f(A_1^2 \cap (A_1^1)^c))^c$.

$f(A_2^t) = <Y, f((A_2^2)^c - (A_2^1)^c), f_-(A_2^2 \cap (A_2^1)^c) >$. Where $f_-(A_2^2 \cap (A_2^1)^c) = (f(A_2^2 \cap (A_2^1)^c))^c$. Also we can prove that

$f((A_1^2)^c - (A_1^1)^c) \subseteq f((A_2^2)^c - (A_2^1)^c)$ and $f_-(A_2^2 \cap (A_2^1)^c) \supseteq f_-(A_1^2 \cap (A_1^1)^c)$

$\Rightarrow f((A_1^2)^c - (A_1^1)^c) \subseteq f((A_2^2)^c - (A_2^1)^c)$.

Therefore $A_1^t \subseteq A_2^t \Rightarrow f(A_1^t) \subseteq f(A_2^t)$

(b) Given $B_1^t \subseteq B_2^t$, where $B_1^t = <X, (B_1^2)^c - (B_1^1)^c, B_1^2 \cap (B_1^1)^c >$. $B_2^t = <X, (B_2^2)^c - (B_2^1)^c, B_2^2 \cap (B_2^1)^c >$

To Prove: $f^{-1}(B_1^t) \subseteq f^{-1}(B_2^t)$

By definition $f^{-1}(B_1^t) = <X, f^{-1}((B_1^2)^c - (B_1^1)^c), f^{-1}(B_1^2 \cap (B_1^1)^c) >$

$f^{-1}(B_2^t) = <X, f^{-1}((B_2^2)^c - (B_2^1)^c), f^{-1}(B_2^2 \cap (B_2^1)^c) >$. One can very easily prove that $f^{-1}((B_1^2)^c - (B_1^1)^c) \subseteq f^{-1}((B_2^2)^c - (B_2^1)^c)$ and $f^{-1}(B_1^2 \cap (B_1^1)^c) \supseteq f^{-1}(B_2^2 \cap (B_2^1)^c)$.

hence $B_1^t \subseteq B_2^t \Rightarrow f^{-1}(B_1^t) \subseteq f^{-1}(B_2^t)$.

(c) To prove $A^* \subseteq f^{-1}(f(A^*))$ and if $f$ is injective.

To prove: $A^* \subseteq f^{-1}(f(A^*))$.

$(A^2)^c - (A^1)^c \subseteq f^{-1}(f((A^2)^c - (A^1)^c))$ and $A^2 \cap (A^1)^c \subseteq f^{-1}(f_-(A^2 \cap (A^1)^c))$ (By lemma 2.10)

Hence $A^* \subseteq f^{-1}(f(A^*))$. 
If \( f \) is injective then
\[
f^{-1}(f(A^*)) \subseteq f^{-1}(\langle X, (A^2)^c - (A^1)^c, (A^2 \cap (A^1)^c) \rangle)
\]
\[
\subseteq f^{-1}(\langle Y, f((A^2)^c - (A^1)^c), f_-(A^2 \cap (A^1)^c) \rangle)
\]
\[
= \langle X, f^{-1}(f((A^2)^c - (A^1)^c)), f^{-1}(f_-(A^2 \cap (A^1)^c)) \rangle
\]

Hence \( f^{-1}(f(A^*)) = \langle X, (A^2)^c - (A^1)^c, (A^2 \cap (A^1)^c) \rangle \)
\[
= A^*.
\]

(d) \( f(f^{-1}(B^*)) \subseteq B^* \) and if \( f \) is onto, then \( f(f^{-1}(B^*)) = B^* \)
\[
f(f^{-1}(B^*)) = f(f^{-1}(\langle Y, (B^2)^c - (B^1)^c, (B^2 \cap (B^1)^c) \rangle))
\]
\[
= f(\langle X, f^{-1}(B^2)^c - (B^1)^c, f^{-1}(B^2 \cap (B^1)^c) \rangle)
\]
\[
f(f^{-1}(B^*)) = \langle Y, f^{-1}(B^2)^c - (B^1)^c, f_-(f^{-1}(B^2 \cap (B^1)^c)) \rangle
\]
\[
\subseteq \langle Y, (B^2)^c - (B^1)^c, (B^2 \cap (B^1)^c) \rangle
\]
\[
= B^*
\]

Notice that
\[
f(f^{-1}(B^2)^c - (B^1)^c) \subseteq (B^2)^c - (B^1)^c
\]
\[
f_-(f^{-1}(B^2 \cap (B^1)^c)) = Y - f(X - f^{-1}(B^2 \cap (B^1)^c))
\]
\[
= Y - f(f^{-1}(Y) - f^{-1}(B^2 \cap (B^1)^c))
\]
\[
= Y - f(f^{-1}(Y - (B^2 \cap (B^1)^c)))
\]
\[
\supseteq Y - (Y - (B^2 \cap (B^1)^c))
\]
\[
= B^2 \cap (B^1)^c
\]

\[
f_-(f^{-1}(B^2 \cap (B^1)^c)) \supseteq B^2 \cap (B^1)^c
\]

(e) To prove \( f^{-1}(\cup B_j^*) = \cup(f^{-1}(B_j)^*) \)
\[
f^{-1}(\cup B_j) = f^{-1}(\langle Y, \cup B_j^1 \cap B_j^2 \rangle)
\]
\[
f^{-1}(\cup B_j^*) = f^{-1}(\langle Y, (\cup B_j^2)^c - (\cup B_j^1)^c, (\cap B_j^2) \cap (\cup B_j^1)^c \rangle)
\]
\[
= \langle X, f^{-1}((\cap B_j^2)^c - (\cup B_j^1)^c), f^{-1}((\cap B_j^2) \cap (\cup B_j^1)^c) \rangle
\]
\[
= \langle X, \cup(f^{-1}(B_j^2)^c - f^{-1}(B_j^1)^c), \cap(f^{-1}(B_j^2) \cap f^{-1}(B_j^1)^c) \rangle
\]
\[
= \cup f^{-1}(\langle Y, (B_j^2)^c - (B_j^1)^c, (B_j^2) \cap (B_j^1)^c \rangle
\]
\[
= \cup(f^{-1}(B_j)^*)
\]

Therefore \( f^{-1}(\cup B_j^*) = \cup(f^{-1}(B_j)^*) \)

(f) We need \( f^{-1}(\cap B_j^*) = \cap(f^{-1}(B_j)^*) \)
\[
f^{-1}(\cap B_j) = f^{-1}(\langle Y, \cap B_j^1 \cup B_j^2 \rangle)
\]

Now, \( f^{-1}(\cap B_j^2) = f^{-1}(\langle Y, (\cup B_j^2)^c - (\cap B_j^1)^c, (\cup B_j^2) \cap (\cap B_j^1)^c \rangle)
\]
\[
= \cap f^{-1}Y, f^{-1}((B_j)^c - (B_j^1)^c), f^{-1}((B_j^2) \cap (B_j^1)^c) \rangle
\]
\[
= \cap f^{-1}(\langle Y, (B_j^2)^c - (B_j^1)^c, (B_j^2) \cap (B_j^1)^c \rangle
\]
\[
= \cap(f^{-1}(B_j)^*)
Therefore \( f^{-1}(\bigcap B_j)^* = \bigcap (f^{-1}(B_j))^* \)

(g) To prove \( f(\bigcup A_i)^* = \bigcup (f(A_i))^* \)

\[
f(\bigcup A_i) = f(\bigtriangleup X, \bigcup A_i^1, \bigcup A_i^2 \succ)
\]

\[
f(\bigcup A_i^1) = f(\bigtriangleup X, (\bigcup A_i^1)^c - (\bigcup A_i^1)^c, (\bigcup A_i^2) \cap (\bigcup A_i^1)^c \succ)
\]

\[
= \bigtriangleup X, f((\bigcup A_i^2)^c - (\bigcup A_i^1)^c), f_-(\bigcup A_i^2) \cap (\bigcup A_i^1)^c) \succ \ldots \ldots (1)
\]

Also

\[
f((\bigcup A_i^2)^c - (\bigcup A_i^1)^c) = f((\bigcup A_i^2)^c - f(\bigcup A_i^1)^c)
\]

\[
= \bigcup f(A_i^2)^c - \bigcup f(A_i^1)^c
\]

\[
= \bigcup (f(A_i^2)^c - f(A_i^1)^c) \ldots \ldots (1)
\]

\[
f_-(\bigcup A_i^2 \cap (\bigcup A_i^1)^c) = Y - f(X - ((\bigcup A_i^2) \cap (\bigcup A_i^1)^c))
\]

\[
= Y - f(X) + f((\bigcup A_i^2) \cap (\bigcup A_i^1)^c)
\]

\[
= Y - f(X) + f((\bigcup A_i^2) \cap (\bigcup A_i^1)^c)
\]

\[
= Y - f(X) + f_-(\bigcup A_i^2) \cap (\bigcup A_i^1)^c)
\]

\[
= Y - f(X) + f_-(\bigcup A_i^2) \cap (\bigcup A_i^1)^c)
\]

\[
= \bigcup (f(A_i^2) \cap (A_i^1)^c) \ldots \ldots (2)
\]

from (1) and (2) in (1) we get

\[
f(\bigcup A_i^1) = \bigcup (f(A_i)^f)
\]

(h) \( f(\bigcap A_i)^* = \bigcap (f(A_i))^* \)

\[
f(\bigcap A_i) = f(\bigtriangleup X, \bigcap A_i^1, \bigcap A_i^2 \succ)
\]

\[
f(\bigcap A_i^1) = f(\bigtriangleup X, (\bigcap A_i^1)^c - (\bigcap A_i^1)^c, (\bigcap A_i^2) \cap (\bigcap A_i^1)^c \succ)
\]

\[
= \bigtriangleup X, f((\bigcap A_i^2)^c - (\bigcap A_i^1)^c), f_-(\bigcap A_i^2) \cap (\bigcap A_i^1)^c) \succ \ldots \ldots (II)
\]

Notice that

\[
f((\bigcap A_i^2)^c - (\bigcap A_i^1)^c) = f((\bigcap A_i^2)^c - f(\bigcap A_i^1)^c)
\]

\[
= \bigcap f(A_i^2)^c - \bigcap f(A_i^1)^c
\]

\[
= \bigcap (f(A_i^2)^c - f(A_i^1)^c) \ldots \ldots (1)
\]

\[
f_-(\bigcap A_i^2) \cap (\bigcap A_i^1)^c = Y - f(X - ((\bigcap A_i^2) \cap (\bigcap A_i^1)^c))
\]

\[
= Y - f(X) + f((\bigcap A_i^2) \cap (\bigcap A_i^1)^c)
\]

\[
= \bigcup (f(A_i^2) \cap (A_i^1)^c) \ldots \ldots (2)
\]
from (1) and (2) in (I) we get

\[ \begin{align*}
&= \prec f(X), \cap (f(A_2^c)^c) - f(A_1^c)^c \cup (f((A_2^c) \cap (A_1^c))^c) \succ \\
&= \cap \prec f(X), f(A_2^c)^c - f(A_1^c)^c \cup f((A_2^c) \cap (A_1^c))^c \succ \\
&= \cap f(\cap A_1^c)^* = \cap f(A_2)^*
\end{align*} \]

(i) \( f^{-1}(Y^c) = f^{-1} \prec Y, \phi^c - Y^c, \phi \cap Y^c \succ \)

\[ \begin{align*}
&= \prec f^{-1}(Y^c), f^{-1}(\phi^c - Y^c), f^{-1}(\phi \cap Y^c) \succ \\
&= \prec X, X - \phi, \phi \cap \phi \succ \\
&= \prec X, \phi^c - X^c, \phi \cap X^c \succ \\
&= \bar{X}^c,
\end{align*} \]

(ii) \( f^{-1}(\phi^c) = f^{-1} \prec Y, \phi^c - \phi^c, \phi \cap \phi^c \succ \)

\[ \begin{align*}
&= \prec f^{-1}(Y^c), f^{-1}(\phi^c - \phi^c), f^{-1}(Y \cap \phi^c) \succ \\
&= \prec X, \phi - X, X \cap X \succ \\
&= \prec X, \phi^c - \phi^c, \phi \cap \phi^c \succ \\
&= \phi^c,
\end{align*} \]

(k) \( f(\bar{X}^c) = \prec f \prec X, \phi^c - X^c, \phi \cap X^c \succ \)

\[ \prec f(X), f(\phi^c - X^c), f_-(\phi \cap X^c) \succ \cdots \cdots (I) \]

Notice that

\[ f(\phi^c - X^c) = f(\phi^c) - f(X^c) \]

\[ = \phi^c - Y^c \cdots (I) \]

\[ f_-(\phi \cap X^c) = f_-(X) + f(\phi \cap X^c) \]

\[ = Y - f(X) + f(\phi) \cap f(X^c) \]

\[ = Y - f(X) + f(\phi) \cap f(\phi) \]

\[ = f(\phi) \cap f(\phi) \cdots (2) \]

from (1) and (2) in (I) we get

\[ \begin{align*}
&= \prec f(X), \phi^c - Y^c, f(\phi) \cap f(\phi) \succ \\
&= \prec f(X), \phi^c - Y^c, \phi \cap \phi \succ \\
&= \prec f(X), \phi^c - Y^c, \phi \cap Y^c \succ \\
&= \bar{Y}^c
\end{align*} \]

(l) \( f(\bar{\phi}^c) = \bar{\phi}^c \)

\[ f(\bar{\phi}^c) = f \prec X, X^c - \phi^c, X \cap \phi^c \succ \]

\[ = \prec f(X), f(X^c - \phi^c), f_-(X \cap \phi^c) \succ \]

\[ = \prec Y, Y^c - \phi^c, Y \cap \phi^c \succ = \bar{\phi}^c \]
Notice that
\[
f(X^c - \phi^c) = f(\phi - X) = f(\phi) - f(X) = \phi - Y = Y^c - \phi^c
\]
\[
f_(X \cap \phi^c) = Y - f(X - (X \cap \phi^c))
\[
= Y - f(X) + f(X \cap \phi^c)
\]
\[
= f(X) \cap f(\phi^c)
\]
\[
= Y \cap \phi^c
\]
\[\text{(m)} \ f(\overline{A^*}) = f < X, (A^2)^c - (A^1)^c, A^2 \cap (A^1)^c >=
\]
\[
= f < X, A^2 \cap (A^1)^c, (A^2)^c - (A^1)^c >=
\]
\[
= f(X), f(A^2 \cap (A^1)^c), f_-(((A^2)^c - (A^1)^c)) >=
\]
\[
= Y, f(A^2 \cap (A^1)^c), f_-(((A^2)^c - (A^1)^c)) >=
\]
\[
f(A^*) = f < X, (A^2)^c - (A^1)^c, A^2 \cap (A^1)^c >
\]
\[
= Y, f_-(A^2 \cap (A^1)^c), f_-(((A^2)^c - (A^1)^c)) >\]..............(I)

since \( f \) is onto and \( f(\overline{A^*}) \subseteq f(A^*) \)

\( f_-(A^2 \cap (A^1)^c) \subseteq f(A^2 \cap (A^1)^c) \) and

\( Y - f(X - (A^2 \cap (A^1)^c)) \subseteq f(A^2 \cap (A^1)^c) \)

\( Y - f(X) + f_-(((A^2 \cap (A^1)^c)) \subseteq f(A^2 \cap (A^1)^c) \)

\( f((A^2 \cap (A^1)^c)) \subseteq f(A^2 \cap (A^1)^c) \)..............(1)

\( f((A^2)^c - (A^1)^c) \supseteq f_-(((A^2)^c - (A^1)^c)) \)

\( f((A^2)^c - (A^1)^c) \supseteq Y - f(X - (A^2)^c - (A^1)^c) \)

\( f((A^2)^c - (A^1)^c) \supseteq Y - f(X) + f_-(((A^2)^c - (A^1)^c)) \)

\( f((A^2)^c - (A^1)^c) \supseteq f((A^2)^c - (A^1)^c)) \)..............(2)

from (1) and (2) in (I) we get

\( f(\overline{A^*}) = f < X, (A^2)^c - (A^1)^c, A^2 \cap (A^1)^c >\)

\[\text{(n)} \ f^{-1}(\overline{B^*}) = f^{-1} < Y, (B^2)^c - (B^1)^c, B^2 \cap (B^1)^c >=
\]
\[
= f^{-1} < Y, B^2 \cap (B^1)^c, (B^2)^c - (B^1)^c >=
\]
\[
= f^{-1}(Y), f^{-1}((B^2)^c - (B^1)^c) >=
\]
\[
= f^{-1}(B^2 \cap (B^1)^c), f^{-1}((B^2)^c - (B^1)^c) >=
\]
\[
f^{-1}(\overline{B^*}) = f^{-1} < Y, (B^2)^c - (B^1)^c, B^2 \cap (B^1)^c >=
\]
\[
= f^{-1}(Y), f^{-1}((B^2)^c - (B^1)^c) >=
\]
\[
= f^{-1}(B^2 \cap (B^1)^c), f^{-1}((B^2)^c - (B^1)^c) >=
\]
\[
f^{-1}(\overline{B^*}) = f^{-1}(\overline{B^*})
III. STAR INTUITIONISTIC TOPOLOGICAL SPACES

Now we generalize the concept of "Star intuitionistic topological space" by means of Star intuitionistic sets: In this case the pair \((X, \tau)\) is always known as an intuitionistic topological space and any set in \(\tau\) is known as an intuitionistic open set in \(X\).

**Definition 3.1.** Let \((X, \tau)\) be an IS topological space. Let \(A_i^* = \langle X, (A_i^0)^c - (A_i^1)^c, A_i^0 \cap (A_i^1)^c \rangle >\) be a star IS set with \(A_i \in \tau\) 

Then \(\tau^* = \{\phi^*, \bar{\tau}^*, A_i^*\}\) is called as the star IS-topological space.

**Example 3.2.** Let \(X = \{a, b, c, d, e\}\) with the topology \(\tau = \{\phi, X, A_1, A_2, A_3, A_4\}\) 

where \(A_1 = \langle X, \{a, b, c\}, \{d\} \rangle, A_2 = \langle X, \{c, d\}, \{e\} \rangle, A_3 = \langle X, \{c\}, \{d, e\} \rangle, A_4 = \langle X, \{a, b, c, d\}, \{\phi\} \rangle\). 

Then \((X, \tau)\) is an intuitionistic topological spaces in \(X\).

We define \(A^* = \langle X, (A^0)^c - (A^1)^c, (A^0) \cap (A^1)^c \rangle >\) and \(\tau^* = \{\phi^*, \bar{\tau}^*, A_i^*, A_i^0, A_i^1\}\) 

where \(A_i^* = \langle X, \{a, b, c\}, \{d\} \rangle, A_i^2 = \langle X, \{c, d\}, \{e\} \rangle, A_i^3 = \langle X, \{c\}, \{d, e\} \rangle, A_i^4 = \langle X, \{a, b, c, d\}, \{\phi\} \rangle\). 

Then \((X, \tau^*)\) is an StarITS on \(X\).

**Definition 3.3.** Let \((X, \tau)\) be a ITS and \(\tau = \{\phi, X, G_i^* : i \in J\}\) 

Then we construct two StarITS’s on \(X\) as follows: 

(a) \(\tau_1^* = \{\phi^*, \bar{\tau}^*, A_i^* \} \cup \{\langle X, \phi^*, G_i^* \rangle : i \in J\}\). 

(b) \(\tau_2^* = \{\phi^*, \bar{\tau}^*, A_i^* \} \cup \{\langle X, (G_i^*)^c - \phi^*, G_i^* \cap \phi^* \rangle : i \in J\}\). 

**Proposition 3.4.** Let \((X, \tau)\) be a Intuitionistic topological space on \(X\). Then we can also construct several ITS’s on \(X\) in the following way: 

(a) \(\tau_{0.1}^* = \{(G^*)^c : \tau \in \tau^*\}\) (b) \(\tau_{0.2}^* = \{\langle \phi^*, G^* \rangle : G^* \in \tau^*\}\).

**Remark 3.5.** Let \((X, \tau^*)\) be a StarITS. 

(a) \(\tau_1^* = \{(G^0)^c - (G^1)^c : \langle X, (G^0)^c - (G^1)^c, G^0 \cap (G^1)^c \rangle \in \tau^*\}\) is a topological space on \(X\).

Similarly \(\tau_2^* = \{G^2 \cap (G^1)^c : \langle X, (G^2)^c - (G^1)^c \rangle \in \tau^*\}\) is a family of all closed sets of the topological space \(\tau^* = \{(G^2)^c - (G^1)^c : \langle X, (G^2)^c - (G^1)^c \rangle \in \tau^*\}\) on \(X\).

(b) Since \((G^2)^c - (G^1)^c \cap G^2 \cap (G^1)^c = \phi\) for each \(G^* = \langle X, (G^2)^c - (G^1)^c, G^2 \cap (G^1)^c \rangle \in \tau^*\), we obtain \((G^2)^c - (G^1)^c \subseteq (G^2 \cap (G^1)^c)\) and \(G^2 \cap (G^1)^c \subseteq ((G^2)^c - (G^1)^c)^c\).

**Example 3.6.** Let \((X, \tau^*)\) be a StarITS .Let \(X = \{a, b\}\) and consider the family 
\(\tau^* = \{\phi^*, \bar{\tau}^*, A^*, B^*\}\) where \(A^* = \langle X, \phi, \{a\} \rangle, B^* = \langle X, \phi, \{b\} \rangle, \phi^* = \langle X, \phi, X \rangle, \bar{\tau}^* = \langle X, X, \phi \rangle\). Then \(\tau_1^* = \{\phi : \langle X, \phi, \{a\} \rangle \in \tau^*\}\) is a topological space on \(X\).

Similarly \(\tau_2^* = \{\{a\} : \langle X, \phi, \{a\} \rangle \in \tau^*\}\) is the family of all closed sets of the topological space.
\[ \tau_2^* = \{ \{a\}^c : \langle X, \phi, \{a\} \rangle^c \in \tau^* \} \] on \( X \)

(b) Since \( \phi \cap \{a\} = \phi \) for each \( G^* = \langle X, \phi, \{a\} \rangle \in \tau^* \), we obtained

\[
\phi \subseteq \{a\}^c \\
\phi \subseteq \{b\} \text{ and} \\
\{a\} \subseteq \{\phi\}^c \\
\{a\} \subseteq \{a, b\}
\]

Hence we conclude that \( (X, \tau_1^*, \tau_2^*) \) is a bitopological space.

**Definition 3.7.** The complement \( \tilde{A}^* \) of an Star IOS \( A^* \) in an ITS \( (X, \tau) \) is called an Star ICS in \( X \). Now we define closure and interior operations in StarITS’s.

**Definition 3.8.** Let \( (X, \tau) \) be an ITS and \( A = \langle X, A^1, A^2 \rangle \) be an IS in \( X \).

Then the interior and closure of \( A \) are defined by

Let \( (X, \tau) \) be an ITS \( A^* = \langle X, (A^2)^c - (A^1)^c, (A^2) \cap (A^1)^c \rangle \) be an IS in \( X \).

Then the int and cl of \( A \) are defined by

\[ Cl(A^*) = \bigcap \{ K^* : K^* \text{ is an Star ICS in } X \text{ and } A^* \subseteq K^* \} \]

\[ int(A^*) = \bigcup \{ G^* : G^* \text{ is an Star IOS in } X \text{ and } G^* \subseteq A^* \} \]

It can be shown that \( Cl(A^*) \) is an StarICS and \( int(A^*) \) is an StarIOS in \( X \), and \( A^* \) is an StarICS in \( X \) iff \( Cl(A^*) = A^* \) and \( A \) is an StarIOS in \( X \) iff \( int(A^*) = A^* \).

**Example 3.9.** Consider the Star ITS \( (X, \tau) \) in Examples 3.2. If \( B^* = \langle X, \{a, c\}, \{d\} \rangle \), then we can write down

\[ int(B^*) = \langle X, \{c\}, \{d, e\} \rangle \text{ and } Cl(B^*) = \langle X, X, \phi \rangle \]

**Proposition 3.10.** Let \( (X, \tau) \) be an StarITS and \( A, B \) be IS’s in \( X \). Then the following properties hold:

(a) \( int(A^*) \subseteq A^* \)

(a^1) \( A \subseteq cl(A^*) \)
(b) $A \subseteq B \Rightarrow \text{int}(A^*) \subseteq \text{int}(B^*)$

(b') $A \subseteq B \Rightarrow \text{Cl}(A^*) \subseteq \text{Cl}(B^*)$

(c) $\text{int}(\text{int}(A^*)) = \text{int}(A^*)$

(c') $\text{cl}(\text{Cl}(A^*)) = \text{Cl}(A^*)$

(d) $\text{int}(A^* \cap B^*) = \text{int}(A^*) \cap \text{int}(B^*)$

(d') $\text{cl}(A^* \cap B^*) = \text{cl}(A^*) \cap \text{cl}(B^*)$

(e) $\text{int}(X^*) = \overline{X}$

(e') $\text{cl}(\overline{X}) = \overline{X}$

**REFERENCES**


