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A Queueing system incorporating the effects of Environmental change and Catastrophes

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Abstract: *In this paper, a limited capacity queueing system incorporating the effects of environmental change and catastrophes is studied. The effect of environmental change is taken to be a function of the number present in the system. We undertake the transient analysis of a limited capacity queueing system with two environmental states in the presence of catastrophes. Transient solution of the queueing model is obtained by using the probability generating function technique. Some interesting particular cases of the queueing model with and without catastrophes are obtained. Measures of effectiveness and steady state solutions of the model are also discussed.*
Keywords: *Transient analysis, Catastrophes, Environment, Finite capacity, Probability generating function.*

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I. Introduction:

From the very beginning, the M/M/1 queue has been the object of systematic and through investigations. In recent years, the attention has been focused to study the queueing systems on certain extensions that include the effect of catastrophes. This consists of adding to the standard assumptions the hypothesis that the number of customers is instantly reset to zero at certain random times. The catastrophes occur at the service- facility as a Poisson process with rate ξ. Whenever a catastrophe occurs at the system, all the customers there are destroyed immediately, the server gets inactivated momentarily, and the server is ready for service when a new arrival occurs. In this connection, a special reference may be made to the paper by A. Di Crescenzo et al. [2].

The notion of catastrophe played a very important role in various areas of science and technology. A large number of research papers have been published on population processes under the influence of catastrophes; see, for instance, P. J. Brockwell [11,12], P. J. Brockwell, J. Gani and S. I. Resnick [13], E. G. Kyriakidis [5] and R. J. Swift [14], among others have discussed birth and death models with catastrophes. These papers are also concerned with various quantities of interest, such as time to extinction. It is also well known that computer networks with a virus may be modeled by queueing networks with catastrophes [15].

It has been proved by A. Di Crescenzo et al. [2] that the M/M/1 catastrophized processes may be suitable to approach a current hot topic of great biological relevance, concerning the interaction between myosin heads and actin filaments that is responsible for force generation during muscle contraction. However, the force of contraction may rise on changing other conditions like a change in temperature or pH or a slight stretching of the fiber. In this paper, we have added another factor of environmental change, i.e. the change in the environment affects the state of the queueing system. In other words, the state of the queueing system is a function of environmental change factors.

By means of a new and extremely sophisticated instrumentation approach by K. Kitamura et al. [9], it has been possible to prove that the sliding of a myosin head over the actin filament occurs by randomly distributed minuscule steps eventually followed by a sudden reset (catastrophe) with an approximately exponentially distributed dwell times. B. Kumar and D. Arivudainambi [3] obtained the transient solution of M/M/1 queueing model with the possibility of catastrophes at the service station. N.K. Jain and D.K. Kanethia [10] studied the transient analysis of a queue with environmental and catastrophic effects.

In this paper, we undertake the analysis of a queueing system in the presence of catastrophes and environmental change in order to obtain some analytical results. In section 2, we have made the assumptions and

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definitions of the model. The detailed analysis of the main model is done in section 3 and some particular cases are obtained in section 4. In section 5 & 6, we have obtained the steady-state result and mean queue length. Applications of the model are discussed in section 7.

II. Assumptions and Definitions:

(i) The customers arrive in the system one by one in accordance with a Poisson process at a single service station. The arrival pattern is non-homogeneous, i.e. there may exists two arrival rates, namely λ_1 and 0 of which only one is operative at any instant.

(ii) The customers are served one by one at the single channel. The service time is exponentially distributed. Further, corresponding to arrival rate λ_1 the Poisson service rate is μ_1 and the service rate corresponding to the arrival rate 0 is μ_2. The state of the system when operating with arrival rate λ_1 and service rate μ_1 is designated as E whereas the other with arrival rate 0 and service rate μ_2 is designated as F.

(iii) The Poisson rate d_n at which the system goes from environmental state E to F tends to decrease or increase whereas at the same time the Poisson rate b_n at which the system moves from environmental state F to E tends to increase or decrease according as the numbers in the queue (say n) increase or decrease from some fixed number (say N). We therefore define,

$$d_n = \beta\left[1+\varepsilon'(N-n)\right] \text{ with } n \le N + \frac{1}{\varepsilon'}$$

and $0 \le n \le N + \dfrac{1}{\varepsilon'} \le M$

Also

$$b_n = \alpha\left[1+\varepsilon(n-N)\right] \text{ with } n \ge N - \frac{1}{\varepsilon}$$

and $0 \le N - \dfrac{1}{\varepsilon} \le n \le M$

Where M denotes the size of the waiting space and ε, ε' are positive numbers such that $\varepsilon \ge \dfrac{1}{N}$ and $\varepsilon' \ge \dfrac{1}{M-N}$. These restrictions on M also are necessary to avoid the negative values of d_n and b_n. When n=N or ε=0, b_n gives the normal rate as α and when n=N or $\varepsilon = \varepsilon'$=0, d_n and b_n gives the normal rates as β and α.

(iv) When the system is not empty, catastrophes occur according to a Poisson process with rate ξ. The effect of each catastrophe is to make the queue instantly empty. Simultaneously, the system becomes ready to accept the new customers.

(v) The queue discipline is first- come-first-served.

(vi) The capacity of the system is limited to M. i.e., if at any instant there are M units in the queue then the units arriving at that instant will not be permitted to join the queue, it will be considered lost for the system.

Define,

$P_n(t)$ = Joint probability that at time t the system is in state E and n units are in the queue, including the one in service.

$Q_n(t)$ = Joint probability that at time t the system is in state F and n units are in the queue, including the one in service.

$R_n(t)$ = The probability that at time t there are n units in the queue, including the one in service.

Obviously,

$$R_n(t) = P_n(t) + Q_n(t)$$

Let us reckon time t from an instant when there are zero customers in the queue and the system is in the environmental state E so that the initial conditions associated with $P_n(t)$ and $Q_n(t)$ becomes,

$$P_n(0) = \begin{cases} 1 & ; \quad n = 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$Q_n(0) = 0$; for all n.

III. Formulation of Model and Analysis (Time Dependent Solution):

The differential-difference equations governing the system are:

$$\frac{d}{dt}P_0(t) = -(\lambda_1 + d_0 + \xi)P_0(t) + \mu_1 P_1(t) + b_0 Q_0(t) + \xi\sum_{n=0}^{M} P_n(t)\,; \; n = 0 \qquad \text{..... (1)}$$

$$\frac{d}{dt}P_n(t) = -(\lambda_1 + \mu_1 + d_n + \xi)P_n(t) + \mu_1 P_{n+1}(t) + \lambda_1 P_{n-1}(t) + b_n Q_n(t)\,;$$

$$0 < n < M \qquad \text{..... (2)}$$

$$\frac{d}{dt}P_M(t) = -(\mu_1 + d_M + \xi)P_M(t) + \lambda_1 P_{M-1}(t) + b_M Q_M(t)\,; \; n = M \qquad \text{.... (3)}$$

$$\frac{d}{dt}Q_0(t) = -(b_0 + \xi)Q_0(t) + \mu_2 Q_1(t) + d_0 P_0(t) + \xi\sum_{n=0}^{M} Q_n(t)\,; \; n = 0 \qquad \text{.... (4)}$$

$$\frac{d}{dt}Q_n(t) = -(\mu_2 + b_n + \xi)Q_n(t) + \mu_2 Q_{n+1}(t) + d_n P_n(t)\,; \; 0 < n < M \qquad \text{.... (5)}$$

$$\frac{d}{dt}Q_M(t) = -(\mu_2 + b_M + \xi)Q_M(t) + d_M P_M(t)\,; \; n = M \qquad \text{.... (6)}$$

Define, the Laplace Transform as

$$\text{L.T. } [f(t)] = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s) \qquad \text{.....(7)}$$

Now, taking the Laplace transforms of equations (1)–(6) and using the initial conditions, we get

$$(s + \lambda_1 + d_0 + \xi)\bar{P}_0(s) - 1 = \mu_1 \bar{P}_1(s) + b_0 \bar{Q}_0(s) + \xi\sum_{n=0}^{M}\bar{P}_n(s) \qquad \text{.... (8)}$$

$$(s + \lambda_1 + \mu_1 + d_n + \xi)\bar{P}_n(s) = \mu_1 \bar{P}_{n+1}(s) + \lambda_1 \bar{P}_{n-1}(s) + b_n \bar{Q}_n(t) \qquad \text{.... (9)}$$

$$(s + \mu_1 + d_M + \xi)\bar{P}_M(s) = \lambda_1 \bar{P}_{M-1}(s) + b_M \bar{Q}_M(s) \qquad \text{.... (10)}$$

$$(s + b_0 + \xi)\bar{Q}_0(s) = \mu_2 \bar{Q}_1(s) + d_0 \bar{P}_0(s) + \xi\sum_{n=0}^{M}\bar{Q}_n(s) \qquad \text{.... (11)}$$

$$(s + \mu_2 + b_n + \xi)\bar{Q}_n(s) = \mu_2 \bar{Q}_{n+1}(s) + d_n \bar{P}_n(s) \qquad \text{.... (12)}$$

$$(s + \mu_2 + b_M + \xi)\bar{Q}_M(s) = d_M \bar{P}_M(s) \qquad \text{.... (13)}$$

Define, the probability generating functions

$$P(z,s) = \sum_{n=0}^{M}\bar{P}_n(s)z^n \qquad \text{.... (14)}$$

$$Q(z,s) = \sum_{n=0}^{M}\bar{Q}_n(s)z^n \qquad \text{.... (15)}$$

$$R(z,s) = \sum_{n=0}^{M}\bar{R}_n(s)z^n \qquad \text{.... (16)}$$

where

$\overline{R}_n(s) = \overline{P}_n(s) + \overline{Q}_n(s)$

Multiplying equations (8)–(10) by the suitable powers of z, summing over all n and using equations (14)–(16), we have.

$$\beta\varepsilon' z^2 P'(z,s) + \alpha\varepsilon z^2 Q'(z,s) + \left[\lambda_1 z^2 - z\{s + \lambda_1 + \mu_1 + \xi + \beta(1+\varepsilon' N)\} + \mu_1\right]P(z,s)$$

$$+ \alpha(1-\varepsilon N) z Q(z,s) = \lambda_1 z^{M+1}(z-1)\overline{P}_M(s) + \mu_1(1-z)\overline{P}_0(s) - z - \xi z \sum_{n=0}^{M} \overline{P}_n(s) \quad \text{.... (17)}$$

Similarly, from equations (11)–(13) and using (14)–(16), we have

$$\beta\varepsilon' z^2 P'(z,s) + \alpha\varepsilon z^2 Q'(z,s) - \beta(1+\varepsilon' N) z P(z,s) + \left[z\{s + \mu_2 + \xi + \alpha(1-\varepsilon N)\} - \mu_2\right]Q(z,s)$$

$$= \mu_2(z-1)\overline{Q}_0(s) + \xi z \sum_{n=0}^{M} \overline{Q}_n(s) \quad \text{.... (18)}$$

Subtracting equation (18) from (17), we have.

$$\left[\lambda_1 z^2 - z(s + \lambda_1 + \mu_1 + \xi) + \mu_1\right]P(z,s) + \left[\mu_2 - z(s + \mu_2 + \xi)\right]Q(z,s) = \lambda_1 z^{M+1}(z-1)\overline{P}_M(s)$$

$$+ \mu_1(1-z)\overline{P}_0(s) - \mu_2(z-1)\overline{Q}_0(s) - z - \xi z \sum_{n=0}^{M} \overline{P}_n(s) - \xi z \sum_{n=0}^{M} \overline{Q}_n(s) \quad \text{.... (19)}$$

Differentiating equation (19) with respect to z, we have

$$\left[\lambda_1 z^2 - z(s + \lambda_1 + \mu_1 + \xi) + \mu_1\right]P'(z,s) + \left[2\lambda_1 z - (s + \lambda_1 + \mu_1 + \xi)\right]P(z,s)$$

$$+ \left[\mu_2 - z(s + \mu_2 + \xi)\right]Q'(z,s) - (s + \mu_2 + \xi)Q(z,s) = \lambda_1 z^{M}\left[(M+2)z - (M+1)\right]\overline{P}_M(s)$$

$$- \mu_1 \overline{P}_0(s) - \mu_2 \overline{Q}_0(s) - 1 - \xi \sum_{n=0}^{M} \overline{P}_n(s) - \xi \sum_{n=0}^{M} \overline{Q}_n(s) \quad \text{.... (20)}$$

Eliminating Q'(z,s) and Q(z,s) from equations (18), (19) and (20), we arrive at a computationally convenient equation.

$$P'(z,s) + \frac{\eta_1(z)}{\eta_2(z)} P(z,s) = \frac{1}{\eta_2(z)}\left[z_1 + z_2\overline{Q}_0(s) + z_3\overline{P}_0(s) + z_4\overline{P}_M(s) + z_5\sum_{n=0}^{M}\overline{P}_n(s) + z_6\sum_{n=0}^{M}\overline{Q}_n(s)\right] \quad \text{.... (21)}$$

where

$$\eta_1(z) = a_1 z^4 + a_2 z^3 + a_3 z^2 + a_4 z + a_5$$

$$\eta_2(z) = z^2\left[\mu_2 - z(s + \mu_2 + \xi)\right]\left[a_6 z^2 + a_7 z + a_8\right]$$

$$\frac{\eta_1(z)}{\eta_2(z)} = \frac{A}{\mu_2 - z(s + \mu_2 + \xi)} + (B + C/z)\frac{1}{z} + \frac{D(2a_6 z + a_7)}{a_6(a_6 z^2 + a_7 z + a_8)}$$

$$+ \frac{(E - a_7 D/2a_6)}{a_6\left[\left(z + \frac{a_7}{2a_6}\right)^2 - \left\{\left(\frac{1}{2}a_7/a_6\right)^2 - \left(\frac{a_8}{a_6}\right)\right\}\right]}$$

$$C = \frac{a_5}{\mu_2 a_8}$$

$$B = \left[a_4 - \frac{a_5}{\mu_2 a_8}\{\mu_2 a_7 - (s + \mu_2 + \xi)a_8\}\right]\frac{1}{\mu_2 a_8}$$

$$A = \begin{vmatrix} (a_3 - b_1 B - b_2 c) & 0 & \mu_2 \\ (a_2 - b_2 B - b_3 c) & \mu_2 & b_4 \\ (a_1 - b_3 B) & b_4 & 0 \end{vmatrix} \frac{1}{\Delta}$$

$$D = \begin{vmatrix} a_8 & \mu_2 & (a_3 - b_1 B - b_2 C) \\ a_7 & b_4 & (a_2 - b_2 B - b_3 C) \\ a_6 & 0 & (a_1 - b_3 B) \end{vmatrix} \frac{1}{\Delta}$$

$$E = \begin{vmatrix} a_8 & 0 & (a_3 - b_1 B - b_2 C) \\ a_7 & \mu_2 & (a_2 - b_2 B - b_3 C) \\ a_6 & b_4 & (a_1 - b_3 B) \end{vmatrix} \frac{1}{\Delta}$$

$$\Delta = \begin{vmatrix} a_8 & 0 & \mu_2 \\ a_7 & \mu_2 & b_4 \\ a_6 & b_4 & 0 \end{vmatrix}$$

$b_1 = \mu_2\, a_7 - a_8\, (s+\mu_2+\xi)$

$b_2 = \mu_2\, a_6 - a_7\, (s+\mu_2+\xi)$

$b_3 = -\, a_6\, (s+\mu_2+\xi)$

$b_4 = -\, (s+\mu_2+\xi)$

$$a_1 = \lambda_1 (s + \mu_2 + \xi)[\alpha\varepsilon + \{s + \mu_2 + \xi + \alpha(1 - \varepsilon N)\}]$$

$$a_2 = -[\ (s + \mu_2 + \xi)\{\beta(1 + \varepsilon' N)(s + \mu_2 + \xi) + \lambda_1\mu_2\} + \{s + \mu_2 + \xi + \alpha(1 - \varepsilon N)\}. \\ \{\lambda_1\mu_2 + (s + \mu_2 + \xi)(s + \lambda_1 + \mu_1 + \xi)\} + 2\lambda_1\varepsilon\, \mu_2\, \alpha\ ]$$

$$a_3 = [\ (s + \mu_2 + \xi)\{2\beta\mu_2(1 + \varepsilon' N) - \alpha\varepsilon\mu_1\} + \mu_2 (s + \lambda_1 + \mu_1 + \xi) \\ \{\alpha\varepsilon + (s + \mu_2 + \xi)\} + \{s + \mu_2 + \xi + \alpha(1 - \varepsilon N)\} \\ \{\mu_1 (s + \mu_2 + \xi) + \mu_2 (s + \lambda_1 + \mu_1 + \xi)\} + (\mu_2 z)^2 \lambda_1\ ]$$

$$a_4 = -[\mu_2^2 \{\beta(1 + \varepsilon' N) + (s + \lambda_1 + \mu_1 + \xi)\} + \mu_1\mu_2 \\ \{s + \mu_2 + \xi + \alpha(1 - \varepsilon N) + (s + \mu_2 + \xi)\}\ ]$$

$$a_5 = \mu_1 \mu_2^2$$

$$a_6 = -\alpha\varepsilon\lambda_1^2$$

$$a_7 = \alpha\varepsilon\, (s + \lambda_1 + \mu_1 + \xi) - \beta\varepsilon'\, (s + \mu_2 + \xi)$$

$$a_8 = (\beta\, \varepsilon'\, \mu_2 - \alpha\, \varepsilon\, \mu_1)$$

$$z_1 = [-z^3 (s + \mu_2 + \xi)[\alpha\varepsilon + \{s + \mu_2 + \xi + \alpha(1 - \varepsilon N)\}]$$

$$+z^2\mu_2\left[2(s+\mu_2+\xi)+\alpha(1-\varepsilon N)+\alpha\varepsilon\right]-z\mu_2^2$$

$$z_2=z(z-1)\left[\mu_2\{2(s+\mu_2+\xi)+\alpha(1-\varepsilon N)\}-z(s+\mu_2+\xi)\cdot\{s+\mu_2+\xi+\alpha(1-\varepsilon N)\}\right]-\mu_2(z-1)\left[\mu_2^2-\{\mu_2-z(s+\mu_2+\xi)\}^2\right]$$

$$+\mu_2\varepsilon\alpha z^2\{\mu_2-z(s+\mu_2+\xi)\}$$

$$z_3=\left[\mu_1\mu_2\{2(s+\mu_2+\xi)+\alpha(1-\varepsilon N)\}z(z-1)+\mu_1(s+\mu_2+\xi)z^2(1-z).\{s+\mu_2+\xi+\alpha(1-\varepsilon N)\}+(1-z)\mu_1\mu_2^2+\mu_1 z^2\alpha\varepsilon\cdot\{\mu_2-(s+\mu_2+\xi)z\}\right]$$

$$z_4=z^M\left[z^3(z-1)\lambda_1(s+\mu_2+\xi)\{s+\mu_2+\xi+\alpha(1-\varepsilon N)\}-z^2(z-1)\lambda_1\mu_2\{2(s+\mu_2+\xi)+\alpha(1-\varepsilon N)\}+z(z-1)\lambda_1\mu_2^2-\alpha\varepsilon z^2\lambda_1\{z(M+2)-(M+1)\}\{\mu_2-z(s+\mu_2+\xi)\}\right]$$

$$z_5=z^3\xi(s+\mu_2+\xi)\left[(s+\mu_2+\xi)-\{s+\mu_2+\xi+\alpha(1-\varepsilon N)\}\right]+z^2\mu_2\xi\left[\alpha\varepsilon+\{s+\mu_2+\xi+\alpha(1-\varepsilon N)\}-(s+\mu_2+\xi)\right]$$

$$z_6=-z^3\xi(s+\mu_2+\xi)\{s+\mu_2+\xi+\alpha(1-\varepsilon N)\}+z^2\mu_2\xi\left[\alpha\varepsilon+(s+\mu_2+\xi)+\{s+\mu_2+\xi+\alpha(1-\varepsilon N)\}\right]-z\mu_2{}^2\xi$$

On solving equation (21), we have

$$P(z,s)=\frac{L_1(z)+L_2(z)\overline{Q}_0(s)+L_3(z)\overline{P}_0(s)+L_4(z)\overline{P}_M(s)+L_5(z)\sum_{n=0}^{M}\overline{P}_n(s)+L_6(z)\sum_{n=0}^{M}\overline{Q}_n(s)}{L(z)}\quad\ldots(22)$$

where

$$L(z)=[\mu_2-z(s+\mu_2+\xi)]^{-A/(s+\mu_2+\xi)}\left[\frac{X(z)-a}{X(z)+a}\right]^{E'}z^B\cdot\left(a_6z^2+a_7z+a_8\right)^{D'}\exp^{(-C/2)}$$

$$D'=\frac{D}{2a_6}$$

$$E'=\left(E-\frac{a_7}{2a_6}D\right)\frac{1}{2a_6}$$

$$X(z)=z+\frac{a_7}{2a_6}$$

$$a=\left[\left(\frac{1}{2}\frac{a_7}{a_6}\right)^2-\left(\frac{a_8}{a_6}\right)\right]^{1/2}$$

$$L_j(z)=\int_0^z\frac{z_j}{\eta_2(z)}L(z)\,dz;\qquad j=1,2,3,4,5,6.$$

Now, from equations (19) and (22), we have

$$Q(z,s)=\frac{L_7(z)+\overline{Q}_0(s)L_8(z)+\overline{P}_0(s)L_9(z)+\overline{P}_M(s)L_{10}(z)+\sum_{n=0}^{M}\overline{P}_n(s)L_{11}(z)+\sum_{n=0}^{M}\overline{Q}_n(s)L_{12}(z)}{B(z)L(z)}\quad\ldots(23)$$

where

$$L_7(z)=L_1(z)\,g(z)-z\,L(z)$$

$$L_8(z)=L_2(z)\,g(z)-\mu_2(z-1)\,L(z)$$

$$L_9(z)=L_3(z)\,g(z)-\mu_1(z-1)\,L(z)$$

$$L_{10}(z)=L_4(z)\,g(z)+\lambda_1 z^{M+1}(z-1)\,L(z)$$

$$L_{11}(z)=L_5(z)\,g(z)-\xi z\,L(z)$$

$$L_{12}(z)=L_6(z)\,g(z)-\xi z\,L(z)$$

$$g(z)=\left[z(s+\lambda_1+\mu_1+\xi)-\mu_1-\lambda_1 z^2\right]$$

$$B(z)=\left[\mu_2-z(s+\mu_2+\xi)\right]$$

Adding equations (22) and (23), we have

$$R(z,s)=\frac{C_1(z)+C_2(z)\overline{Q}_0(s)+C_3(z)\overline{P}_0(s)+C_4(z)\overline{P}_M(s)+C_5(z)\sum_{n=0}^{M}\overline{P}_n(s)+C_6(z)\sum_{n=0}^{M}\overline{Q}_n(s)}{B(z)\,L(z)} \quad .... (24)$$

where

$C_i(z) = B(z)\,L_i(z) + L_{i+6}(z)$; i=1, 2, 3, 4, 5, 6.

Since,

$$R(1,s)=\sum_{n=0}^{M}\overline{R}_n(s)=\frac{1}{s} \quad .... (25)$$

Thus equation (24) for z=1, gives

$$R(1,s)=\frac{1}{s}=\lim_{z\to 1}R(z,s) \quad .... (26)$$

$$P(0,s)=\overline{P}_0(s)=\lim_{z\to 0}P(z,s) \quad .... (27)$$

And $$Q(0,s)=\overline{Q}_0(s)=\lim_{z\to 0}Q(z,s) \quad .... (28)$$

The equations (26), (27) and (28) on solution gives the values of $\overline{P}_0(s)$, $\overline{Q}_0(s)$, $\overline{P}_M(s)$, $\sum_{n=0}^{M}\overline{P}_n(s)$ and $\sum_{n=0}^{M}\overline{Q}_n(s)$.

Again, we have from equations (22) and (23) on setting z=1 and $\overline{P}_0(s) = P_0$, $\overline{Q}_0(s) = Q_0$, $\overline{P}_M(s) = P_M$, $\sum_{n=0}^{M}\overline{P}_n(s) = P_n$ and $\sum_{n=0}^{M}\overline{Q}_n(s) = Q_n$

$$P(1,s)=\frac{L_1(1)+L_2(1)Q_0+L_3(1)P_0+L_4(1)P_M+L_5(1)\sum_{n=0}^{M}P_n+L_6(1)\sum_{n=0}^{M}Q_n}{L(1)} \quad .... (29)$$

$$Q(1,s)=\frac{L_7(1)+L_8(1)Q_0+L_9(1)P_0+L_{10}(1)P_M+L_{11}(1)\sum_{n=0}^{M}P_n+L_{12}(1)\sum_{n=0}^{M}Q_n}{B(1)\,L(1)} \quad .... (30)$$

These on inversions give the respective probabilities for the system to be in the environmental states E and F.

IV. Particular Cases:

Case I Setting n=N or $\varepsilon=0$ in equations (22) and (23), (i.e., when the rate of change of environment from state F to E is constant), we have

$$P(z,s)=\frac{L_1'(z)+L_2'(z)\overline{Q}_0(s)+L_3'(z)\overline{P}_0(s)+L_4'(z)\overline{P}_M(s)+L_5'(z)\sum_{n=0}^{M}\overline{P}_n(s)+L_6'(z)\sum_{n=0}^{M}\overline{Q}_n(s)}{L'(z)} \quad ...(31)$$

$$Q(z,s)=\frac{L_7'(z)+L_8'(z)\overline{Q}_0(s)+L_9'(z)\overline{P}_0(s)+L_{10}'(z)\overline{P}_M(s)+L_{11}'(z)\sum_{n=0}^{M}\overline{P}_n(s)+L_{12}'(z)\sum_{n=0}^{M}\overline{Q}_n(s)}{B'(z)\,L'(z)} \quad ...(32)$$

where

$$L_i'(z)=L_i(z)\big|_{\varepsilon=0} \quad ; \ i=1, 2, 3, \dots 12.$$

$$L'(z)=L(z)\big|_{\varepsilon=0}$$

$$B'(z)=B(z)\big|_{\varepsilon=0}$$

On adding equations (31) and (32), we have.

$$R(z,s)=\frac{C_1'(z)+C_2'(z)\overline{Q}_0(s)+C_3'(z)\overline{P}_0(s)+C_4'(z)\overline{P}_M(s)+C_5'(z)\sum_{n=0}^{M}\overline{P}_n(s)+C_6'(z)\sum_{n=0}^{M}\overline{Q}_n(s)}{B'(z)\,L'(z)} \quad ... (33)$$

where

$$C_i'(z)=B'(z)L_i'(z)+L_{i+6}'(z) \ ; \ i=1, 2, 3, 4, 5, 6.$$

The unknown quantities $\overline{Q}_0(s)$, $\overline{P}_0(s)$, $\overline{P}_M(s)$, $\sum_{n=0}^{M}\overline{P}_n(s)$ and $\sum_{n=0}^{M}\overline{Q}_n(s)$ can be evaluated as before.

Case II Setting $\varepsilon=\varepsilon'=0$ or n=N in equation (17) and (18), (i.e. when the rates of interchange of environmental states from E to F and F to E is constant), we have

$$X_1(z)P(z,s)+X_2(z)Q(z,s)+X_3(z)=0 \quad .... (34)$$

$$X_4(z)P(z,s)+X_5(z)Q(z,s)+X_6(z)=0 \quad .... (35)$$

where

$$X_1(z)=-\left[\lambda_1 z^2-z\,(s+\lambda_1+\mu_1+\beta+\xi)+\mu_1\right]$$

$$X_2(z)=-\alpha z$$

$$X_3(z)=-\left[\mu_1(z-1)\,\overline{P}_0(s)+\lambda_1 z^{M+1}(1-z)\overline{P}_M(s)+z+\xi z\sum_{n=0}^{M}\overline{P}_n(s)\right]$$

$$X_4(z)=\beta z$$

$$X_5(z)=\left[\mu_2-z(s+\mu_2+\alpha+\xi)\right]$$

$$X_6(z)=\left[\mu_2(z-1)\overline{Q}_0(s)+\xi z\sum_{n=0}^{M}\overline{Q}_n(s)\right]$$

From equations (34) and (35), we have

$$P(z,s)=\frac{X_2(z)X_6(z)-X_3(z)X_5(z)}{X_1(z)X_5(z)-X_2(z)X_4(z)} \quad .... (36)$$

$$Q(z,s)=\frac{X_4(z)X_3(z)-X_1(z)X_6(z)}{X_1(z)X_5(z)-X_2(z)X_4(z)} \quad .... (37)$$

Thus, we have

$$R(z,s)=\frac{\begin{array}{l}\mu_2(z-1)[X_2(z)-X_1(z)]\bar{Q}_0(s)+\xi z\sum_{n=0}^{M}\bar{Q}_n(s)[X_2(z)-X_1(z)]+\mu_1(1-z)\\ [X_4(z)-X_5(z)]\bar{P}_0(s)+\lambda_1 z^{M+1}[X_5(z)-X_4(z)](1-z)\bar{P}_M(s)+\\ z[X_5(z)-X_4(z)]+\xi z\sum_{n=0}^{M}\bar{P}_n(s)[X_5(z)-X_4(z)]\end{array}}{-z^2s^2+sX_7(z)+(1-z)X_8(z)-z^2\xi(\alpha+\beta+\xi)} \quad \text{.... (38)}$$

where

$$X_7(z)=\lambda_1 z^3-z^2(\lambda_1+\mu_1+\mu_2+\alpha+\beta+2\xi)+z(\mu_1+\mu_2)$$

$$X_8(z)=-z^2\lambda_1(\alpha+\mu_2+\xi)+z[\alpha\mu_1+\mu_2(\lambda_1+\mu_1+\beta+\xi)]-\mu_1\mu_2.$$

And $P(1,s)=\sum_{n=0}^{M}\bar{P}_n(s)=\frac{s+\alpha+\xi}{s(s+\alpha+\beta+\xi)}$

$$Q(1,s)=\sum_{n=0}^{M}\bar{Q}_n(s)=\frac{\beta}{s(s+\alpha+\beta+\xi)}$$

Relation (38) is a polynomial in z and exists for all values of z, including the three zeros of the denominator. Hence $\bar{P}_0(s)$, $\bar{Q}_0(s)$ and $\bar{P}_M(s)$ are obtained by setting the numerator equal to zero and substituting the three zeros, α_1, α_2 and α_3 (say) of the denominator (at each of which the numerator must vanish).

Now letting $\alpha\to\infty$, $\beta\to 0$ and setting $\mu_1=\mu_2=\mu$ (say) in relation (38), we have

$$r(z,s)=\frac{(1-z)\mu\bar{R}_0(s)-(1-z)\lambda_1 z^{M+1}\bar{P}_M(s)-z-\xi z/s}{\lambda_1 z^2-z(s+\lambda_1+\mu+\xi)+\mu} \quad \text{.... (39)}$$

where

$$\bar{R}_0(s)=\bar{P}_0(s)+\bar{Q}_0(s)$$

$$r(z,s)=\lim_{\beta\to 0}\left[\lim_{\alpha\to\infty}R(z,s)\right]$$

Relation (39) is a polynomial in z and exists for all values of z, including the two zeros of the denominator. Hence, $\bar{R}_0(s)$ and $\bar{P}_M(s)$ can be evaluated as before.

Case III Putting $\varepsilon=\varepsilon'=1$, N=1 in equation (24), (i.e. when $d_n=\beta n$ and $b_n=\alpha n$), we have.

$$R(z,s)=\frac{C_1''(z)+\bar{Q}_0(s)C_2''(z)+\bar{P}_0(s)C_3''(z)+\bar{P}_M(s)C_4''(z)+\sum_{n=0}^{M}\bar{P}_n(s)C_5''(z)+\sum_{n=0}^{M}\bar{Q}_n(s)C_6''(z)}{B(z)L''(z)} \quad \text{... (40)}$$

where

$$L''(z)=L(z)\big|_{\varepsilon=\varepsilon'=1,\,N=1}$$

$$C_i''(z)=C_i(z)\big|_{\varepsilon=\varepsilon'=1,\,N=1};\quad i=1, 2, 3, 4, 5, 6$$

V. Steady State Results:

This can at once be obtained by the well-known property of the Laplace transform given below:

$\lim_{t\to\infty} f(t)=\lim_{s\to 0} s\,\bar{f}(s)$, If the limit on the left hand side exists.

Thus if

$$R(z)=\sum_{n=0}^{M} R_n \; z^n$$

Then

$$R(z)=\lim_{s\to 0} s \; R\;(z,s)$$

By using this property, we have from equation (24) for the steady state

$$R(z)=\frac{N_1(z)Q_0+N_2(z)P_0+N_3(z)P_M+N_4(z)\sum_{n=0}^{M}P_n+N_5(z)\sum_{n=0}^{M}Q_n+N}{B^*(z)L^*(z)} \qquad \text{.... (41)}$$

where

$$R_n = \lim_{s\to 0} s\,\overline{R}_n(s)$$

$$N_i(z)=B(z)L^*_{i+1}(z)+L^*_{i+7}(z)\Big|_{s=0} \; ; \; i=1, 2, 3, 4, 5$$

$$B^*(z)=B(z)\big|_{s=0}$$

$$L^*(z)=L(z)\Big|_{s=0}$$

$$L_i^*(z)=\int \frac{z_i}{\eta_2(z)}L(z)\,dz \quad ; \; i=1, 2, 3, 4, 5, 6$$

$$L_8^*(z)=L_2^*(z)\,g(z)-\mu_2(z-1)L(z)$$

$$L_9^*(z)=L_3^*(z)\,g(z)-\mu_1(z-1)L(z)$$

$$L_{10}^*(z)=L_4^*(z)\,g(z)+\lambda_1\,z^{M+1}(z-1)L(z)$$

$$L_{11}^*(z)=L_5^*(z)\,g(z)-\xi z\,L(z)$$

$$L_{12}^*(z)=L_6^*(z)\,g(z)-\xi z\,L(z)$$

and N = The constant of integration.

The unknown quantities P_0, $Q_{0,}$ P_M, $\sum_{n=0}^{M}P_n$ and $\sum_{n=0}^{M}Q_n$ can be evaluated as before.

Particular cases:

Case I Relation (33), on applying the theory of Laplace transform, we have

$$R(z)=\frac{Q_0\,N_1'(z)+N_2'(z)P_0+N_3'(z)P_M+N_4'(z)\sum_{n=0}^{M}P_n+N_5'(z)\sum_{n=0}^{M}Q_n+N'}{T_1(z)\,T_2(z)} \qquad \text{.... (42)}$$

where,

$$N_i'(z)=B'(z)L_{i+1}^{**}(z)+L_{i+7}^{**}(z)\Big|_{s=0} \qquad ; \; i=1, 2, 3, 4, 5.$$

$$T_1(z)=B'(z)\big|_{s=0}$$

$$L_j^{**}(z)=\int\left[\frac{z_j}{\eta_2(z)}L(z)\right]_{\epsilon=0} dz \qquad ; \; j=2, 3, 4, 5, 6.$$

$$L_8^{**}(z) = L_2^{**}(z)\ g(z) - \mu_2(z-1)L'(z)$$

$$L_9^{**}(z) = L_3^{**}(z)\ g(z) - \mu_1(z-1)L'(z)$$

$$L_{10}^{**}(z) = L_4^{**}(z)\ g(z) + \lambda_1 z^{M+1}(z-1)L'(z)$$

$$L_{11}^{**}(z) = L_5^{**}(z)\ g(z) - \xi z\, L'(z)$$

$$L_{12}^{**}(z) = L_6^{**}(z)\ g(z) - \xi z\, L'(z)$$

N' = the constant of integration.

The unknown quantities Q_0, P_0, P_M, $\sum_{n=0}^{M} P_n$ and $\sum_{n=0}^{M} Q_n$ can be evaluated as before.

Case II Relation (38), on applying the theory of Laplace transforms gives

$$R(z) = \frac{\begin{array}{l}\mu_2(1-z)\left\{\alpha z + z(\lambda_1+\mu_1+\beta+\xi) - \lambda_1 z^2 - \mu_1\right\}Q_0 + \\ \mu_1(1-z)[\beta z - \{\mu_2 - z(\mu_2+\alpha+\xi)\}]P_0 + \lambda_1 z^{M+1}(1-z)\{\mu_2 - z(\mu_2+\alpha+\xi) - \beta z\}P_M + \\ \xi z/(\alpha+\beta+\xi)\left[\beta\left\{\lambda_1 z^2 - z(\lambda_1+\mu_1+\alpha+\beta+\xi)+\mu_1\right\} + (\alpha+\xi)\{\mu_2 - z(\mu_2+\alpha+\beta+\xi)\}\right]\end{array}}{\begin{array}{l}z^3\lambda_1(\mu_2+\alpha+\xi) - z^2[\lambda_1(\mu_2+\alpha+\xi) + \{\alpha\mu_1+\mu_2(\lambda_1+\mu_1+\beta+\xi)\} + \xi(\alpha+\beta+\xi)] \\ + z[\{\alpha\mu_1+\mu_2(\lambda_1+\mu_1+\beta+\xi)\} + \mu_1\mu_2] - \mu_1\mu_2\end{array}} \quad \text{.... (43)}$$

or, we can write

$$R(z) = \frac{T(z)Q_0 + N(z)P_0 + L(z)P_M + M(z)}{K(z)} \quad \text{.... (44)}$$

Where T(z), N(z) and L(z) are the co-efficient of Q_0, P_0 and P_M respectively in the numerator of equation (43) and K(z) is the denominator of (43).

Equation (44) is a polynomial in z and exists for all values of z, including three zeros of the denominator. Hence Q_0, P_0 and P_M can be obtained by setting the numerator equal to zero. Substituting the three zeros b_1, b_2 and b_3 (say) of the denominator (at each of which the numerator must vanish).

Three equations determining the constants Q_0, P_0 and P_M are

$$T(b_1)Q_0 + N(b_1)P_0 + L(b_1)P_M = -M(b_1) \quad \text{.... (45)}$$

$$T(b_2)Q_0 + N(b_2)P_0 + L(b_2)P_M = -M(b_2) \quad \text{.... (46)}$$

$$T(b_3)Q_0 + N(b_3)P_0 + L(b_3)P_M = -M(b_3) \quad \text{.... (47)}$$

After solving these equations, we have

$$Q_0 = \frac{-M(b_1)A_{11} + M(b_2)A_{21} - M(b_3)A_{31}}{A}$$

$$P_0 = \frac{M(b_1)A_{12} - M(b_2)A_{22} + M(b_3)A_{32}}{A}$$

$$P_M = \frac{-M(b_1)A_{13} + M(b_2)A_{23} - M(b_3)A_{33}}{A}$$

where

$$A=\begin{vmatrix} T(b_1) & N(b_1) & L(b_1) \\ T(b_2) & N(b_2) & L(b_2) \\ T(b_3) & N(b_3) & L(b_3) \end{vmatrix}$$

A_{ij} is the co-factor of the $(i, j)^{th}$ element of A.

By putting the values of Q_0, P_0 and P_M in equation (44), we have

$$R(z)=\frac{T(z)[-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}]+N(z)[M(b_1)A_{12}-M(b_2)A_{22}+M(b_3)A_{32}] +L(z)[-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}]+A\cdot M(z)}{A\cdot K(z)} \quad \text{.... (48)}$$

VI. Mean Queue Length:

Define,

L_q= Expected number of customers in the queue including the one in service.

Then

$L_q = R'(z)\big|_{z=1}$

Therefore, from equation (48), we have

$$L_q=\frac{\begin{array}{l}K(1)[T'(1)\{-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}\}+N'(1)\{M(b_1)A_{12}-M(b_2)A_{22}+M(b_3)A_{32}\} \\ +L'(1)\{-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}\}+A\,M'(1)]-[T(1)\{-M(b_1)A_{11}+M(b_2)A_{21} \\ -M(b_3)A_{31}\}+N(1)\{M(b_1)A_{12}-M(b_2)A_{22}+M(b_3)A_{32}\}+L(1)\{-M(b_1)A_{13}+M(b_2)A_{23} \\ -M(b_3)A_{33}\}+A\cdot M(1)]\,K'(1)\end{array}}{A\cdot[K(1)]^2}$$

.... (49)

where dashes denotes the first derivative w. r. t. z.

Relation (39), on applying the theory of Laplace transforms gives

$$r(z)=\frac{(1-z)\mu R_0-(1-z)\lambda_1 z^{M+1}P_M-\xi z}{\lambda_1 z^2-z(\lambda_1+\mu+\xi)+\mu} \quad \text{.... (50)}$$

where

$$r(z)=\lim_{s\to 0} s\,r(z,s)$$

Equation (50) is a polynomial in z and exists for all values of z, including the two zeros of the denominator. Hence R_0 and P_M can be obtained by setting the numerator equal to zero. Substituting the two zeros a_1 and a_2 (say) of the denominator (at each of which the numerator must vanish).

Two equations determining the constants R_0 and P_M are

$$(1-a_1)\mu R_0-(1-a_1)\lambda_1 a_1^{M+1}P_M=\xi a_1 \quad \text{.... (51)}$$

$$(1-a_2)\mu R_0-(1-a_2)\lambda_1 a_2^{M+1}P_M=\xi a_2 \quad \text{.... (52)}$$

On solving these equations, we have

$$P_M=\frac{(a_1-a_2)}{a_1^{M+1}-a_2^{M+1}} \quad \text{and} \quad R_0=\frac{a_2\,\xi}{(1-a_2)\mu}+\frac{\lambda_1}{\mu}a_2^{M+1}\frac{a_1-a_2}{a_1^{M+1}-a_2^{M+1}} \;;$$

where

$$\lambda_1(1-a_1)\,(1-a_2)=-\xi,\quad a_1>1$$

Now, from equation (50), we have

$$r(z)=\frac{\xi+\lambda_1(1-z)(1-a_2)\dfrac{(a_1-a_2)}{a_1^{M+1}-a_2^{M+1}}\ \dfrac{a_2^{M+1}-z^{M+1}}{a_2-z}}{\lambda_1(z-a_1)(a_2-1)} \quad .... (52)$$

r_n = The co-efficient of z^n

$$r_n=\frac{a_2}{\mu(1-a_2)}\left[\xi(1-P_M)+a_2^{M-n}\,\frac{a_1^{n+1}-a_2^{n+1}}{a_1-a_2}\right]\left(\frac{\lambda_1}{\mu}\right)^n a_2^n \quad .... (54)$$

If $\xi=0$ (i.e., no catastrophe is allowed), then from equation (50), we have

$$r(z)=\frac{\mu R_0-\lambda_1 z^{M+1}P_M}{\mu-\lambda_1 z} \quad .... (55)$$

The condition, $\lim_{z\to 1} r(z)=1$ gives

$$\mu R_0-\lambda_1 P_M=\mu-\lambda_1 \quad .... (56)$$

As r(z) is analytic, the numerator and denominator of equation (55) must vanish simultaneously for $z=\mu/\lambda_1$, which is a zero of its denominator. Equating the numerator of equation (55) to zero for $z=\mu/\lambda_1$ we have

$$R_0=\rho^{-M}P_M, \qquad \rho=\lambda_1/\mu<1 \quad .... (57)$$

Relation (56) and (57) gives

$$R_0=\frac{1-\rho}{1-\rho^{M+1}}\ ,\quad P_M=\frac{(1-\rho)\rho^M}{1-\rho^{M+1}}$$

Now, from equation (55), we have

$$r(z)=\frac{1-\rho}{1-\rho^{M+1}}\ .\ \left[\frac{1-(\rho z)^{M+1}}{1-\rho z}\right] \quad .... (58)$$

which is a well known result of the M/M/1 queue with finite waiting space M.

When there is an infinite waiting space, the corresponding expression for r(z) is obtained by letting M tends to infinity in equation (58), If $(\rho,|z|)<1$.

$$r(z)=\frac{1-\rho}{1-\rho z} \quad .... (59)$$

which is again a well known result of the M/M/1 queue with infinite waiting space.

Case III Relation (40), on applying the theory of Laplace Transform gives

$$R(z)=\frac{H_1(z)Q_0+H_2(z)P_0+H_3(z)P_M+H_4(z)\sum_{n=0}^{M}P_n+H_5(z)\sum_{n=0}^{M}Q_n+H'}{B_1(z)\ H(z)} \quad .... (60)$$

where,

$$B_1(z)=B(z)\big|_{s=0}$$

$$H(z)=L''(z)\big|_{s=0}$$

$$H_i(z)=B(z)L''_{i+1}(z)+L''_{i+7}(z)\big|_{s=0} \qquad ;\ i=1, 2, 3, 4, 5.$$

$$L''_j(z)=\int\left[\frac{z_j}{\eta_2(z)}L(z)\right]_{\varepsilon=\varepsilon'=1,\,N=1} dz \qquad ;\ j=2, 3, 4, 5, 6.$$

$$L_k''(z)=L_k(z)\big|_{\varepsilon=\varepsilon'=1,\,N=1} \qquad ;\ k= 8, 9, 10, 11, 12.$$

H' = the constant of Integration.

The unknown quantities of equation (60) can be evaluated as before.

VII. Applications of the model in biological phenomenon & agriculture:

1. There are many creatures such as cockroaches, ants, mosquitoes etc whose movement is restricted with the change of temperature (environment). As the temperature drops below a critical temperature say T_0, the movement (production) of such like creatures becomes almost zero. On the other hand, as the temperature goes higher than T_0 the movement becomes normal. The catastrophes may occur with these creatures in both the environmental states i.e., spray etc which make them zero instantaneously. Then the number of such like creatures present in any area can be estimated by using the described queueing model with environmental change and catastrophes.

2. In agriculture, if a crop is infected with a particular species of insects due to change in temperature (environment), we may use some chemical agents or compounds to treat such type of insects. The number of bacteria that destroys the crop, in large part, relies on the effectiveness and amount of the chemical reagents used. In other words, the use of the chemical reagents can wipe out the whole of the insects or a part of it. The effect of these chemical reagents on bacteria which make them zero instantaneously can be regarded as the occurrence of a catastrophe.

VIII. Conclusion:

In this paper, we have established the effects of environmental change and catastrophes on the limited capacity queueing system. We have obtained some particular cases and steady state solutions. Some measures of effectiveness are also obtained.

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