Some Stronger Forms of supra $bT$- Continuous Functions

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**ABSTRACT:** The aim of this paper is to introduce new classes of functions called strongly supra $bT$-Continuous and perfectly supra $bT$ - continuous functions and study some of their properties and relations among them.

**KEYWORD:** $bT^\alpha$-closed (open) sets, $bT^\alpha$- continuous, $bT^\alpha$- irresolute, strongly $bT^\alpha$- continuous and perfectly $bT^\alpha$- continuous.

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**I. INTRODUCTION**


**II. PRELIMINARIES**

**Definition 2.1[4,6]** A subfamily of $\mu$ of $X$ is said to be a supra topology on $X$, if

(i) $X, \emptyset \in \mu$

(ii) If $A_i \in \mu$ for all $i \in I$ then $\bigcup A_i \in \mu$.

The pair $(X, \mu)$ is called supra topological space. The elements of $\mu$ are called supra open sets in $(X, \mu)$ and complement of a supra open set is called a supra closed set.

**Definition 2.2[6]**

(i) The supra closure of a set $A$ is denoted by $cl^\mu(A)$ and is defined as $cl^\mu(A) = \cap\{B : B$ is a supra closed set and $A \subseteq B\}$.

(ii) The supra interior of a set $A$ is denoted by $int^\mu(A)$ and defined as $int^\mu(A) = \cup\{B : B$ is a supra open set and $A \supseteq B\}$.

**Definition 2.3[4]** Let $(X, \tau)$ be a topological spaces and $\mu$ be a supra topology on X. We call $\mu$ a supra topology associated with $\tau$ if $\tau \subseteq \mu$.

**Definition 2.4[6]** Let $(X, \mu)$ be a supra topological space. A set $A$ is called a supra $b$-open set if $A \subseteq cl^\mu(int^\mu(A)) \cup int^\mu(cl^\mu(A))$. The complement of a supra $b$-open set is called a supra $b$-closed set.

**Definition 2.5[3]** A subset $A$ of a supra topological space $(X, \mu)$ is called $bT^H$-closed set if $bcl^H(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $T^H$ - open in $(X, \mu)$.

**Definition 2.6[3]** Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and $\mu$ be an associated supra topology with $\tau$. A function $f : (X, \tau) \to (Y, \sigma)$ is called $bT^H$-continuous if $f^{-1}(V)$ is $bT^H$- closed in $(X, \mu)$ for every closed set $V$ of $(Y, \sigma)$.

**Definition 2.7[3]** Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and $\mu$ be an associated supra topology with $\tau$. A function $f : (X, \tau) \to (Y, \sigma)$ is called $bT^H$- irresolute if $f^{-1}(V)$ is $bT^H$- closed in $(X, \mu)$ for every $bT^H$- closed set $V$ of $(Y, \sigma)$. 
III. STRONGLY bT*-CONTINUOUS AND PERFECTLY bT*-CONTINUOUS.

Definition 3.1 Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X,τ)→(Y,σ) is called Strongly bT*-Continuous if the inverse image of every bT*-closed in Y is supra closed in X.

Theorem 3.2 Let (X,τ) be a topological spaces and μ be an associated supra topology with τ. A function f:(X,τ)→(Y,σ) is Strongly bT*-Continuous if the inverse image of every bT*-closed set in Y is supra closed in X.

Proof Assume that f is strongly bT*-Continuous. Let G be any supra closed set in Y. By the theorem 3.2[3] every supra closed set is bT*-closed in Y, G is bT*-closed in Y. Since f is strongly bT*-continuous, f⁻¹(G) is supra closed in X. Therefore f is bT*-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3 Let X=Y={a,b,c}, τ={X,ϕ,{a}} and σ={Y,ϕ,{a},a,b}. Let f:(X,τ)→(Y,σ) be an identity map. Then f is bT*-continuous but not strongly bT*-continuous, since for the bT*-closed set V={c} in Y, f⁻¹(V)=f⁻¹({c})={c} is not supra closed in X.

Theorem 3.4 A function f:(X,τ)→(Y,σ) be strongly bT*-continuous if and only if the inverse image of every bT*-closed set in Y is supra closed in X.

Proof Assume that f is strongly bT*-continuous. Let F be any bT*-closed set in Y. Then F is bT*-open set in Y. Since f is strongly bT*-continuous, f⁻¹(F) is supra open in X. But f⁻¹(F) = X-f⁻¹(F) and so f⁻¹(F) is supra closed in X.

Conversely, assume that the inverse image of every bT*-closed set in Y is supra closed in X. Let G be any bT*-open set in Y. Then G is bT*-closed set in Y. By assumption, f⁻¹(G) is supra closed in X. But f⁻¹(G) = X-f⁻¹(G) and so f⁻¹(G) is supra open in X. Therefore f is strongly bT*-continuous.

Theorem 3.5 If a function f:X→Y is strongly bT*-continuous and a map g: Y→Z is bT*-continuous, then the composition gof : X→Z is strongly bT*-continuous.

Proof Let G be any supra closed in Z. Since g is bT*-continuous, g⁻¹(G) is bT*-closed in Y. Since f is strongly bT*-continuous, f⁻¹(g⁻¹(G)) is supra closed in X. But (gof)⁻¹(G) = f⁻¹(g⁻¹(G)). Therefore gof is strongly bT*-continuous.

Theorem 3.6 If a function f: X→Y is strongly bT*-continuous and a map g: Y→Z is bT*-continuous, then the composition gof : X→Z is bT*-continuous.

Proof Let G be any supra closed in Z. Since g is bT*-continuous, g⁻¹(G) is bT*-closed in Y. Since f is strongly bT*-continuous, f⁻¹(g⁻¹(G)) is supra closed in X. By the theorem (3.2)[3] every supra closed set is bT*-closed, f⁻¹(g⁻¹(G)) is bT*-closed. But f⁻¹(g⁻¹(G)) = (gof)⁻¹(G). Therefore gof is bT*-continuous.

Theorem 3.7 If a function X→Y is supra continuous then it is strongly bT*-continuous but not conversely.

Proof Let f: X→Y is supra continuous. Let F be a supra closed set in Y. Since f is continuous, f⁻¹(F) is supra closed in X. By the theorem (3.2)[3] every supra closed set is bT*-closed set, f⁻¹(F) is bT*-closed. Hence f is bT*-continuous.

Converse of the above theorem need not be true as seen from the following example.

Example 3.8 Let X=Y={a,b,c}, τ={X,ϕ,[a,b]} and σ={Y,ϕ,[a,b],[c]}. Let f:(X,τ)→(Y,σ) be an identity map. Then f is Strongly bT*-continuous, V={b} is bT*-closed in Y, f⁻¹(V)=f⁻¹({b})={b} is supra closed in X. Since V={b} is not supra closed in Y, f is not supra continuous.

Definition 3.9 Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X,τ)→(Y,σ) is called perfectly bT*-Continuous if the inverse image of every bT*-closed in Y is both supra closed and supra open in X.
**Theorem 3.10** Let \((X, \tau)\) be a topological spaces and \(\mu\) be an associated supra topology with \(\tau\). A function \(f:(X, \tau)\rightarrow (Y, \sigma)\) is perfectly \(bT^n\)-continuous then it is strongly \(bT^\mu\)-continuous.

**Proof** Assume that \(f\) is perfectly \(bT^n\)-continuous. Let \(G\) be any \(bT^n\)-closed set in \(Y\). Since \(f\) is perfectly \(bT^n\)-continuous, \(f^{-1}(G)\) is supra closed in \(X\). Therefore \(f\) is strongly \(bT^\mu\)-continuous.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.11** Let \(X=Y=\{a,b,c\}\), \(\tau=\{X,\phi,\{a\}\}\) and \(\sigma=\{Y,\phi,\{a\},\{c\},\{a,b\},\{a,c\}\}\). Let \(f:(X, \tau)\rightarrow (Y, \sigma)\) be an identity map. Then \(f\) is strongly \(bT^\mu\)-continuous but not perfectly \(bT^\mu\)-continuous, since for the \(bT^\mu\)-closed set \(V=\{b,c\}\) in \(Y\), \(f^{-1}(V)=f^{-1}((b,c))=\{b,c\}\) is not in both supra open and supra closed in \(X\).

**Theorem 3.12** A function \(f:(X,\tau)\rightarrow (Y,\sigma)\) be perfectly \(bT^\mu\)-continuous if and only if the inverse image of every \(bT^\mu\)-closed set in \(Y\) is both supra open and supra closed in \(X\).

**Proof** Assume that \(f\) is perfectly \(bT^\mu\)-continuous. Let \(F\) be any \(bT^\mu\)-closed set in \(Y\). Then \(F^c\) is \(bT^\mu\)-open set in \(Y\). Since \(f\) is perfectly \(bT^\mu\)-continuous, \(f^{-1}(F^c)\) is both supra open and supra closed in \(X\). But \(f^{-1}(F^c)=X\smallsetminus f^{-1}(F)\) and so \(f^{-1}(F)\) is both supra open and supra closed in \(X\).

Conversely, assume that the inverse image of every \(bT^\mu\)-closed set in \(Y\) is both supra open and supra closed in \(X\). Let \(G\) be any \(bT^\mu\)-open set in \(Y\). Then \(G^c\) is \(bT^\mu\)-closed set in \(Y\). By assumption, \(f^{-1}(G^c)\) is supra closed in \(X\). But \(f^{-1}(G^c)=X\smallsetminus f^{-1}(G)\) and so \(f^{-1}(G)\) is both supra open and supra closed in \(X\). Therefore \(f\) is perfectly \(bT^\mu\)-continuous.

**Theorem 3.13** If a function \(X\rightarrow Y\) is strongly \(bT^\mu\)-continuous then it is \(bT^\mu\)-irresolute but not conversely.

**Proof** Let \(f:X\rightarrow Y\) is strongly \(bT^\mu\)-continuous function. Let \(F\) be a \(bT^\mu\)-closed set in \(Y\). Since \(f\) is strongly \(bT^\mu\)-continuous, \(f^{-1}(F)\) is supra closed in \(X\). By the theorem (3.2)(3) every supra closed set is \(bT^\mu\)-closed set, \(f^{-1}(F)\) is \(bT^\mu\)-closed in \(X\). Hence \(f\) is \(bT^\mu\)-irresolute.

Converse of the above theorem need not be true as seen from the following example.

**Example 3.14** Let \(X=Y=\{a,b,c\}\), \(\tau=\{X,\phi,\{a\}\}\) and \(\sigma=\{Y,\phi,\{a\},\{c\}\}\). Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be an identity map. Then \(f\) is \(bT^\mu\)-irresolute but not strongly \(bT^\mu\)-continuous, since for the \(bT^\mu\)-closed set \(V=\{b\}\) in \(Y\), \(f^{-1}(V)=f^{-1}((b))=\{b\}\) is not supra closed in \(X\).

**Theorem 3.15** If a function \(X\rightarrow Y\) is perfectly \(bT^\mu\)-continuous then it is \(bT^\mu\)-irresolute but not conversely.

**Proof** Let \(f:X\rightarrow Y\) is perfectly \(bT^\mu\)-continuous function. Let \(F\) be a \(bT^\mu\)-closed set in \(Y\). Since \(f\) is perfectly \(bT^\mu\)-continuous, \(f^{-1}(F)\) is both supra open and supra closed in \(X\). By the theorem (3.2)[3] every supra closed set is \(bT^\mu\)-closed set, \(f^{-1}(F)\) is \(bT^\mu\)-closed in \(X\). Hence \(f\) is \(bT^\mu\)-irresolute.

Converse of the above theorem need not be true as seen from the following example.

**Example 3.16** Let \(X=Y=\{a,b,c\}\), \(\tau=\{X,\phi,\{a\}\}\) and \(\sigma=\{Y,\phi,\{a\},\{c\}\}\). Let \(f:(X,\tau)\rightarrow (Y,\sigma)\) be an identity map. Then \(f\) is \(bT^\mu\)-irresolute but not perfectly \(bT^\mu\)-continuous, since for the \(bT^\mu\)-closed set \(V=\{b\}\) in \(Y\), \(f^{-1}(V)=f^{-1}((b))=\{b\}\) is not in both supra open and supra closed in \(X\).

From the above theorem and example we have the following diagram

Supra continuous \(\Downarrow\) Perfectly \(bT^\mu\)-continuous \(\Downarrow\) Strongly \(bT^\mu\)-continuous \(\rightarrow\) \(bT^\mu\)-irresolute \(\Downarrow\) \(bT^\mu\)-continuous
REFERENCES